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MICROTRONS (ELECTRON CYCLOTRONS) FOR X AND K BAND OPERATION

BY

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ABSTRACT

Certain desirable features of the microtron favor its development as a compact, high energy particle accelerator. When scaled for use at higher frequencies increased compactness results. This article describes two X-band microtrons which have been built and successfully operated and gives the experimental data on their operation. The principles involved in scaling the microtron for X and K-band frequencies are discussed.

INTRODUCTION

The principle of the microtron was first published[2] by Veksler (1)[3] in 1944. This principle was stated in such generality that a number of accelerator schemes, including the conventional cyclotron, could be considered within its scope. Since a cavity resonator (endovibrator) was mentioned, an accelerator system with orbits cotangential in the resonator was implied, and this is the arrangement to which the term "microtron" applies in this paper. The microtron idea was reported and published independently by Itoh and Kobayashi (2) (first reported in 1946).

The microtron was first successfully operated in the 10-cm. R.F. band by Henderson (W.J.), LeCaine, and Montalbetti (3) and later described at length by Redhead, LeCaine, and Henderson (W.J.) (4). Recently another 10-cm. microtron was built and operated by Henderson (C.), Heymann, and Jennings (7) at University College, London. First operation of the 3-cm. microtrons described herein has been reported by the author previously (5).

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[2] The microtron principle has been stated independently without publication by others. Alvarez discussed this idea in classroom lectures as early as 1939. Schwinger (6) is also known to have proposed the idea during the last war.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

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THEORY

An electron moving transversely to a uniform magnetic field moves in a circular orbit having a period of orbital rotation given by $T = \frac{2\pi}{Hec} E$ where e, H, c have their usual meanings and E is the total energy of the particle. In Fig. 1 an electron moving in this way is shown passing

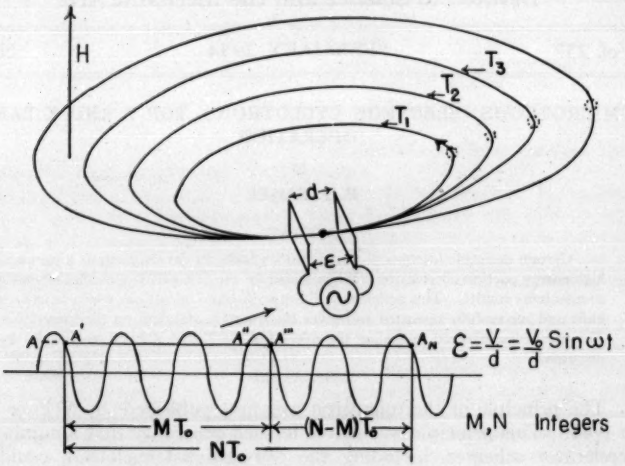

FIG. 1. Microtron principle and nomenclature.

repeatedly through an acceleration gap across which an alternating electric field is acting at constant frequency. Suppose that at phase A of the electric field the electron starts at one side of the gap and in it gains energy of amount c_1E_0 on crossing. It then describes a circular orbit with period

$$T_1 = \frac{2\pi}{Hec}(1 + c_1)E_0, \tag{1}$$

where c_1 is fraction of a rest mass energy $E_0 = m_0c^2$ gained. By proper choice of the frequency ($f_0 = 1/T_0$) this orbital period may be made exactly equal to an integral number M of R.F. periods so that $T_1 = M T_0$. The electron returns to the gap at phase A'' and on crossing gains again some energy c_2E_0. It then describes another circular orbit with period

$$T_2 = \frac{2\pi}{Hec}(1 + c_1 + c_2)E_0 \tag{2}$$

and for proper values of c_2 this orbital period may be another integral number N of R.F. periods, where $N > M$, so that $T_2 = N T_0$. Thus,

it seems that for a given R.F., f_0 and field H, a proper set of values for c_1, c_2, M, and N will insure that the electron repeatedly returns to the gap at a phase suitable for further acceleration. From (1) and (2)

$$\frac{2\pi}{Hec}\,\bar{c}_2E_0 = T_2 - T_1 (= T_3 - T_2 = T_4 - T_3 \ldots) \tag{3}$$

by further calculation[4] assuming for the time being that the particle will gain on the average in subsequent gap crossings an "average energy" $\bar{c}_2E_0$. Gap passage requires a small fraction of the total orbital period and will affect the leaving phase A_n of the particle. Detailed analysis indicates that there is a phase stability region in the vicinity of the R.F. voltage wave peak in which the phase will oscillate about an equilibrium value. Then $\bar{c}_2E_0$ represents the average energy gain per gap crossing, after the first. From (1) and (2) we have

$$\frac{T_2}{T_1} = \frac{N}{M} = 1 + \frac{\bar{c}_2}{1 + c_1} \tag{4}$$

from which

$$\bar{c}_2 = \frac{N - M}{M}\,(1 + c_1). \tag{5}$$

From (4) or (5) it is seen that since N and M are integers (neither equal to zero) then c_1 and $\bar{c}_2$ must be integers or rational fractions, giving the unique feature of the microtron that the average energy gain per electron turn is some simple fraction (or multiple) of the electron's rest mass energy.

A considerable number of possible combinations of N, M, c_1, and $\bar{c}_2$ are possible. In the simplest type of operation $c_1 = \bar{c}_2 = \dfrac{N - M}{2M - N}$. Table I shows the possible combinations of N and M in this case.

TABLE I.

$c_1 = \bar{c}_2$:	2	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\dfrac{N - M}{M}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
N	5	3	4	5	6	7
M	3	2	3	4	5	6

(When $c_1 \neq \bar{c}_2$ a much more complicated table results, having interesting implications which will not be discussed here.)

In the above table the usual value of $N - M$ is 1, but modes in which $N - M$ has other values are obtained by multiplying the lower

[4] For example

$$T_4 - T_3 = \frac{2\pi}{Hec}\,(1 + c_1 + c_2 + c_2' + c_2'') - \frac{2\pi}{Hec}\,(1 + c_1 + c_2 + c_2')$$
$$= \frac{2\pi}{Hec}\,c_2'' \cong \frac{2\pi}{Hec}\,c_2.$$

rows by integers without affecting c_2. Since $N - M$ is one in the simplest cases, modes will be referred to in this paper by the value of c_2 or by c_2E_0: thus 1, $\frac{1}{2}$, $\frac{1}{3}$ . . . or 511, 255, 170, . . . Kev modes.

The phase stability of the microtron has been discussed in several theoretical and experimental papers (1, 2, 4, 7, 8, 9). Redhead, LeCaine, and Henderson (4) treated the phase stability by an approximate differential equation obtaining a phase stable range given by

$$\sin \alpha_n = \frac{d/\lambda}{4 \sin \chi/2}\left(1 + \frac{v_n}{v_{n-1}}\right) \tag{6}$$

where v_n and v_{n-1} are the velocities in the nth and $n - 1$ transits, d is the gap length, λ the R.F. wavelength, and α, χ, angles representing, respectively, $\frac{\omega(t_f + t_i)}{2}$ and $\frac{\omega(t_f - t_i)}{2}$ in which t_f and t_i are, respectively, the times of leaving and entering the gap. Itoh and Kobayashi (2) deduced a similar relation giving about the same angular range. They, and also Henderson, Heymann, and Jennings (7), and Schmelzer (8) have studied the phase stability by the method of plotting successive energy deviations against their preceding phase deviations. All of these have found a range of phase stability agreeing somewhat with that given above and furthermore, have shown that there are considerable ranges of phase excursion in which no final phase stability is reached but in which the particle may still be accelerated successfully because the number of acceleration turns in the microtron is finite. Recently further work on the phase stability for a 10-cm. microtron has been carried out by Kaufmann (9). It is not likely that the subject of phase stability has been completely explored and exhausted.

SCALING AND DIMENSIONING THEORY

The measure of the compactness of an accelerator is the ratio E_f/r_f of final particle energy to the final orbit radius, but since at relativistic energies $E \sim Hr$, this is approximately the strength of the magnetic field as given directly by Eqs. 1 and 3. Thus

$$E_f/r_f = H = \frac{2\pi E_0 (1 + c_1)}{ec \quad MT_0} \equiv \frac{2\pi E_0 c_2}{ec(N - M)T_0} \tag{7}$$

or

$$H = 0.3572 f_0 \frac{c_2}{N - M} \text{ when } f_0 \text{ is in mc/s.} \tag{8}$$

These equations show that maximum compactness is achieved by making c_1, c_2 and f_0 as large as possible and keeping M and $(N - M)$ low (unity if possible).

The energy gain per electron turn c_2E_0 is given by

$$c_2E_0 = e\int_0^d \epsilon d\epsilon = eV' \tag{9}$$

where ($\epsilon = V/d$) and V' is an effective gap voltage which may be written

$$V' = \mathfrak{c}_2 E_0/e = \frac{Hc}{\omega} \quad (N - M). \tag{10}$$

The radius of any electron orbit of total energy E is given by

$$r_n = \frac{1}{He}(E_n^2 - E_0^2)^{\frac{1}{2}} \tag{11}$$

but may be determined more easily from the W.P.A. Table of Electronic Functions (N.B.S.). Using (11) together with (1)

$$H = \frac{E_0}{er_1}(2c_1 + \mathfrak{c}_1^2)^{\frac{1}{2}} = \frac{2\pi E_0}{e}\frac{(1 + c_1)}{M\lambda} \quad (\lambda = cT_0) \tag{12}$$

from which the radius of the first orbit is given by

$$r_1 = \frac{4}{\pi}\frac{(2c_1 + \mathfrak{c}_1^2)^{\frac{1}{2}}}{1 + c_1} M\frac{\lambda}{8} \tag{13}$$

or

$$r_1 = \begin{cases} 1.102\ M\lambda/8 & (c_1 = 1) \\ 0.950\ M\lambda/8 & (c_1 = \frac{1}{2}). \end{cases} \tag{14}$$

The cavity resonator (which not only provides the electric accelerating field but also provides electrons by field emission) must be so dimensioned that the orbit diameter is at least greater than the resonator radius. For $M = 1$ the latter would have to be less than $\lambda/4$ which is physically difficult since for resonance one resonator dimension must be of the order $\lambda/2$. Fortunately $M = 1$ only in certain special cases.

In any scaling it is necessary to consider the possible loss per turn of energy by electron radiation as given by the Schwinger (4, 6) relation:

$$L_n = \frac{4\pi e^2}{3r_n}\left(\frac{E_n}{E_0}\right)^4. \tag{15}$$

This loss becomes appreciable either at very small r or large E_n. Since $E_n \sim Hr$, $L \sim r^3$ and is most important in the final acceleration turns. The value of $\mathfrak{c}_2$ taken together with the maximum permissible phase deviation determines the number of orbits possible.

In Fig. 2 are plotted the magnetic field H, and the compactness figure, expressed as final particle energy at $r = 10$ cm. (an actual orbit at exactly 10 cm. is not implied) against the operating frequency in kilomegacycles. Developments in the microtron field are still on the left side of the diagram and very considerable improvement in the compactness figure is indicated in principle. One may compare the 4.6 Mev final energy obtained by Henderson *et al.* (4) at $r = 15$ cm. in a 1000-gauss field at 2.8 kmc. with the theoretically possible 25 Mev at $r = 10$ cm. at 24 kmc. (assuming the same mode with $c_2E_0 = 511$ Kev).

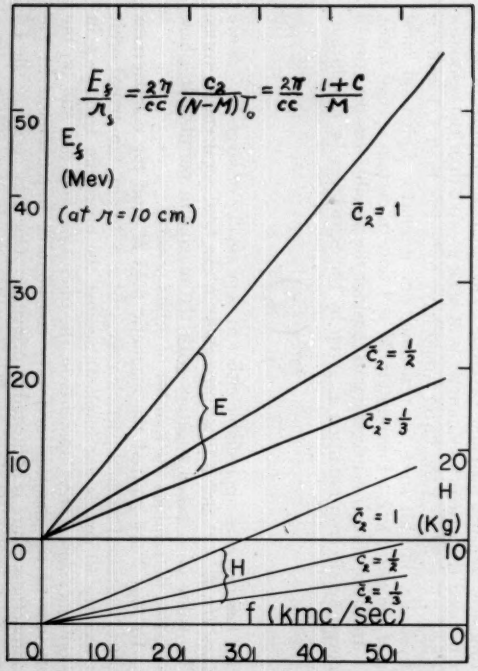
$\frac{E_s}{\pi_s} = \frac{2\pi}{cc}\frac{c_2}{(N-M)\tau_0} = \frac{2\pi}{cc}\frac{1+C}{M}$
E_s
(Mev)
(at π = 10 cm.)
$\bar{C}_2 = 1$
$\bar{C}_2 = \frac{1}{2}$
$\bar{C}_2 = \frac{1}{3}$
E
H
(Kg)
f (kmc/sec)
50
40
30
20
10
0
10
20
0
10
20
30
40
50

FIG. 2.

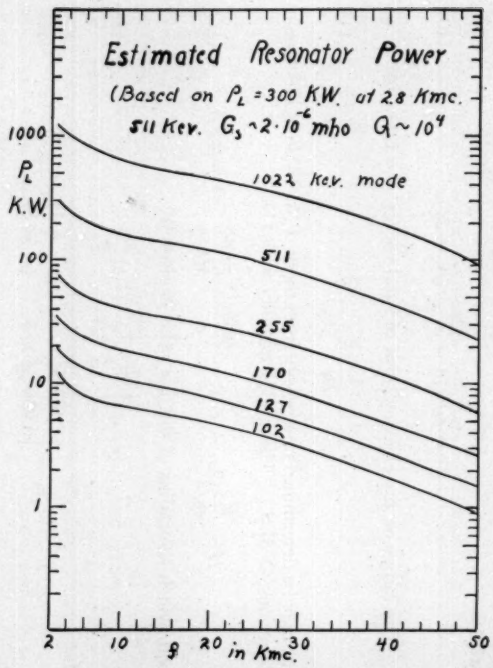
Estimated Resonator Power
(Based on P_L = 300 K.W. at 2.8 Kmc.
511 Kev. $G_3 \sim 2 \cdot 10^{-6}$ mho $Q \sim 10^4$
1000
P_L
K.W.
100
10
1
1022 Kev. mode
511
255
170
127
102
2
10
20
30
40
50
f in Kmc.

FIG. 3.

This will require K band magnetrons giving around 100 KWP, a development likely to be achieved.

In Fig. 3 is shown the estimated resonator peak power required for microtron operation when $N - M = 1$ and c_2E_0 has values from $2E_0$ to $0.2E_0$ (1022 to 102.2 Kev/electron turn). These curves are tentative and only a rough approximation, all being based on a figure of 300 KWP reported by Henderson (4) for 511 Kev operation in the 10-cm. band. The curves are based on the power relation

$$P_L = \frac{V'^2}{2R_s} \tag{16}$$

where V' is the peak gap voltage and R_s the lumped, loaded shunt resistance of the resonator and electron beam. In scaling, the unloaded shunt resistance will vary as $\lambda^{\frac{1}{2}}$ and in the above it is assumed that the effective shunt resistance will vary in about the same way. (It is hoped that direct experimental data on power required for microtron resonators operating at various frequencies and with varied conditions of beam loading will become available as microtron development proceeds. More exact calculations could also be made by making some assumptions on the probable beam loadings to be encountered.) Thus

$$P_L = \frac{V'^2}{2}\left(\frac{1}{R_s} + G_b\right) = \frac{V'^2}{2R_s'} \tag{17}$$

where

$$R_s' = \frac{Q}{\omega C + \dfrac{\text{const.}}{\omega} N} \tag{18}$$

where N is the electron density. This reduces to the ordinary unloaded shunt resistance when N is zero or ω very large.

Although the power curves may be over-optimistic on the high frequency side, they show that power requirements for the lower modes $c_2 = \frac{1}{2}, \frac{1}{4}$ etc., or for $N - M = 2, 3$ etc. are not too severe and at least suggest further development. In the 3-cm. band the indicated power is 165 KWP for $c_2 = 1$ (511 Kev) operation, or 42 KWP for $c_2 = 0.5$ (255 Kev) operation. This is well within possibility with ordinary X-band magnetrons, and borne out by the experience gained to date with two X-band microtrons. While the indicated power at 24 KMC is about 100 KWP for $c_2 = 1$ the lower modes are within possibility, $c_2 = 0.5$ requiring about 25 KWP.

MAGNET STRUCTURE AND FOCUSING

In the microtron the magnetic gap height is determined by the resonator size. This decreases of itself as the operating frequency is raised, but it is advantageous to reduce the gap height as far as possible.

Ideas contemplated are: (1) use of a split or race-track type of magnet to reduce the gap height to about the resonator orifice diameter, (2) encasement of the resonator in a pole face well with means for correction of the disturbance in the magnetic field caused thereby, (3) development or resonators with smaller height, and (4) use of smaller dielectric filled resonators. Despite discouragement with the latter due to losses, this type of resonator has recently been reconsidered for use in linear accelerators (10).

Ordinarily, no magnetic focusing is provided in the microtron except that given by placing the resonator in the fall-off region of the field

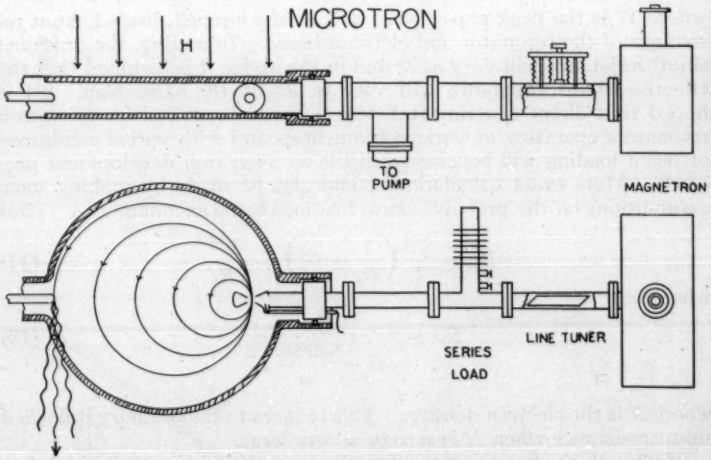

FIG. 4.

(1–2 per cent of the center field). Some particle focusing is given by the electrostatic field of the resonator since the phase-stable region is mainly on the falling voltage side of the voltage peak. In the X-band work it was found important not to increase the resonator orifice beyond a certain value. Although the total electron yield was increased, the orbital stability became poorer.

In view of this it will be of interest to study the effect of built-in magnetic focusing and also correction by electrostatic deflection. Some ideas in this connection have been indicated by Schmelzer (8).

Electron beam removal is easy in principle because of the finite spacing of the orbits and has been accomplished in the 10-cm. microtron of the Henderson (U. Col.) group by using a steel tube to select and shield the outgoing electron beam.

X-BAND MICROTRONS

To test the possibilities of 3-cm. microtron operation, a 1.3-Mev (total energy) unit was first built and studied. This was followed by a second, 3.0-Mev, unit to be further developed as a sealed-off unit. Both microtrons have been powered by a 4-J50 magnetron and experimental pulser described briefly below.

Figure 4 shows the schematic arrangement of a microtron as followed for the 1.3 Mev unit, in which no elaborate design was involved. A brass vacuum chamber with built-in iron pole lids was fitted out as shown and mounted in an available magnet yoke of conveniently small

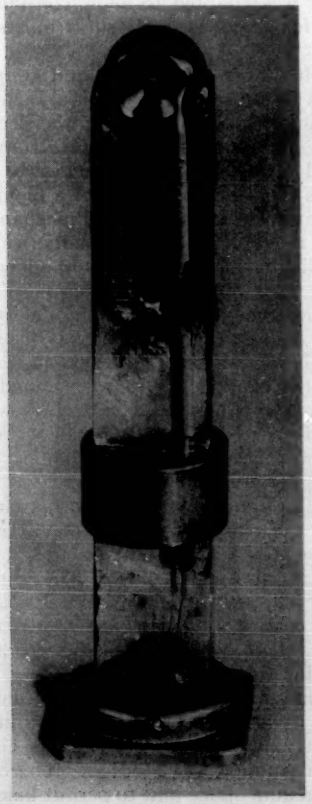

FIG. 5.

FIG. 6.

size. A wave-guide resonator mount was fitted to the vacuum chamber through an O-ring seal joint which allowed both radial positioning and axial tilting of the resonator. This resonator stem was connected by an evacuated R.F. line containing a pump tee, a series load, and a dielectric-slug line tuner, to the 4-J50 magnetron. On the opposite side of the vacuum chamber was mounted another sliding O-ring seal carrying a movable probe rod for orbit testing and target use.

The pulser unit follows in circuit detail the Model 9 (11) magnetron pulser with the exception that a lower power d-c. supply was used and Eimac 450 TL switch tubes were employed in place of the customary Type 2000. The modulator circuit was adapted to allow continuous control of the pulse width and also for producing very short pulses by cable delay.

The resonator, as mounted on its wave-guide stem, and coupled thereto by a circular resonant iris, is shown in Fig. 5, which also shows the small silvered glass tube coaxial line for probe monitoring the electric field of the cavity. Figure 6 shows the opposite side of the resonator mount on which a horseshoe-shaped filament is mounted close to the resonator throat. (No experience is reported on the use of a filament since its need has been absent.) In later work a cavity tuner mechanism consisting of a Pyrex glass plunger, movable in and out of the widest part of the cavity in a direction parallel to the cavity axis, was added to give a tuning range of 2–3 mc/s.

The cavity resonators were first designed according to the theory of Hansen and Richtmyer (12) with confocal ellipsoidal-hyperboloidal inner surfaces, approximated by "sphere and cone" surfaces machined in oxygen-free high conductivity copper blanks. These were joined by silver-hydrogen brazing on their common equator. The cavities were scaled in all dimensions until a nearly correct resonant frequency was obtained and then final adjustment was carried out by mechanical deformation to alter the resonator gap length. Adjustment of the coupling iris hole was made at the same time to obtain both matching and correct frequency for operation on a fixed-frequency magnetron. Typical resonator characteristics were: $Q_0 = 4100$ and $R_s = 1.3 \times 10^6$ ohms. In most of the experiments a pulse width of $0.25\mu s$ was used together with repetition rates up to 1000 pps. From the cavity voltage build-up equation $V = V_0(1 - e^{-\omega t/2Q})$ the voltage is about 98 per cent of maximum at $t = 0.25$ μs for $Q_L = 1878$. Longer pulses were also tried without obtaining any decided improvement in operation.

The construction of the 3.8 Mev microtron is given in Figs. 7 and 8 which show respectively, a sectional view of the interior, and the unit as assembled and connected to the R.F. power source. To obtain compactness and symmetry a circular construction was adopted. To avoid the location of magnet coils in the vacuum space and to permit baking out of the unit without magnet coils in place a circular web of non-

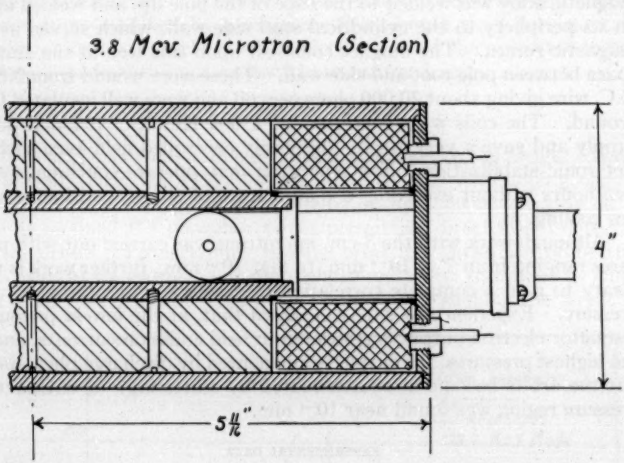

FIG. 7.

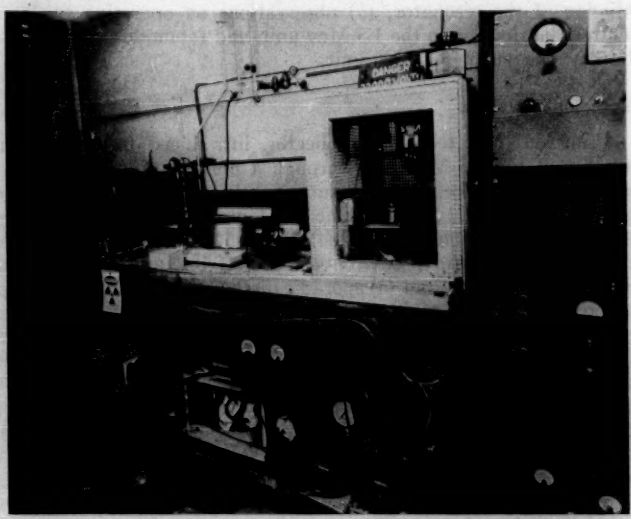

FIG. 8.

magnetic alloy was welded to the edge of the pole lip, and welded again on its periphery to the cylindrical steel side wall, which served as the magnetic return. The magnet coils rest upon this web in the annular space between pole root and side wall. These were wound from No. 34 E.C. wire giving about 20,000 ohms per coil and were well insulated from ground. The coils were energized by a 0–5 Kev d-c. 100 ma. power supply and gave a very steady and easily controlled field even without electronic stabilization. The microtron may operate continuously for two hours without exceeding a temperature of 60° C. and longer with fan cooling.

Although work with the 3-cm. microtrons was carried out with pressures ranging from 2×10^{-6} mm. to 8×10^{-4} mm., further work is necessary to gain a complete correlation of accelerator performance with pressure. Experience to date has shown that, at the lowest pressures, resonator electron output is restricted to field emission currents, and at the highest pressures, the output is augmented by discharge phenomena, but the orbits become more attenuated by scattering. A satisfactory pressure region was found near 10^{-4} mm.

EXPERIMENTAL DATA

Experimental data obtained to date are: (*a*) studies on electron emission of the resonator, (*b*) observations on operating modes and electron distribution in the 1.3 Mev unit, and (*c*) mode and orbit power observations on the 3.8-Mev unit.

Resonator Emission

Using an insulated probe collector, introduced into the vacuum chamber of the 1.3-Mev unit through a port in the direction of the resonator axis, the total electron output of the resonator in zero magnetic field was measured. This current was found to be of two sorts, (*a*) a fairly steady emission current due to true field emission in the intense fields at the resonator throat (of the order of 10^{-7} amp. at threshold of resonator excitation), and (*b*) sporadic discharges reaching many microamperes which seem to be associated with gas discharges or metallic eruptions. To get a better idea of the structure of the emission beam, a photographic technique was adopted. By using a circular step tablet of calibrated Al foils, not only the maximum energies of the electrons but also their distribution in number density may be determined. Film exposure was reduced to a useful figure by gating the modulator for a 10 pulse operation shot. From the results it may be concluded that the emission beam consists of a core of very energetic electrons (K.E. > 300 Kev) of diameter nearly equal to that of the resonator orifice, plus a "halo" of slow electrons (but even higher in number density). For 107-Kev electrons the halo diameter is about five times that of the core and for the 292-Kev electrons (10 steps of 0.001" Al) the halo is about

twice core diameter. A more detailed study of this emission pattern especially as modified by a magnetic field should yield important information on the manner in which electrons leave the resonator throat, and perhaps lead to improvement in resonator design.

Orbit Data from 1.3-Mev Microtron

The first operation of this microtron was observed by Geiger counter indication (Dec. 29, 1951). After some trouble with sliding electric probes, orbit observations were carried on by a probe carrying on its end an inclined P-5 phosphor target permitting direct visual detection of the orbits. To determine relative orbit intensities the probe tube was fitted at its viewing end with a photo-multiplier tube head by which the intensities could be measured. By properly shaping the target tip

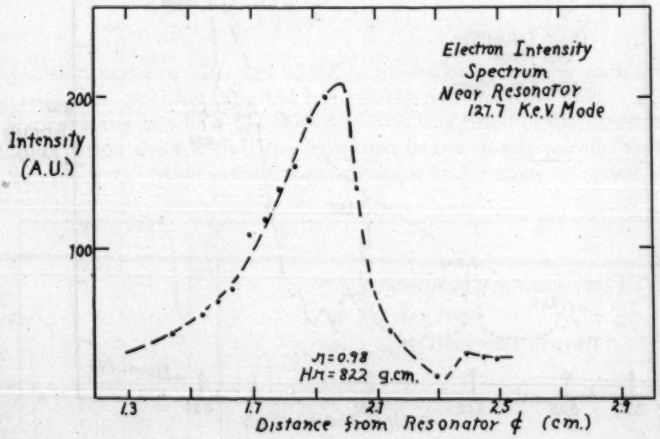

FIG. 9.

and using an entry slit, good resolution, a complete picture of the electron spectrum near the resonator, and the fine structure of the orbits may be obtained.

Microtron orbits could be observed when the magnet current was set to give magnetic fields of 1677, 1118 and 839 gauss corresponding to operation in 255, 170 and 127 Kev ($N - M = 1$) modes. Only one or two 255-Kev orbits could be obtained, four orbits in the 170-Kev mode and five in the 127-Kev mode. The positions of these orbits were found to agree within limits of error with their theoretical positions and the number of orbits in a whole pattern was all that could be expected from known extent of the useful field region. In each mode a strong "orbit"

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was noted close to resonator and was found to be a peak in the electron spectrum. This is shown in Fig. 9 for the 127.7 Kev mode ($c_2 = \frac{1}{4}$). Figure 10 shows the orbit intensities of the 170-Kev mode as measured by photo-probe and as determined visually.

It was noted that the shape of the orbit beams depended very much on the field current setting, being round and sharp in cases such as in Fig. 10 with very little structure between orbits; again, in other cases, distributions resembling Saturnian rings could be observed. Of particular interest was the observation in both 127- and 170-Kev modes that the first true orbit often appears double, the parts being at slightly different radii and different in vertical position. While it is tempting and easy to explain this as operation in a closely neighboring mode, the fact that this is not observed further out indicates that this explanation

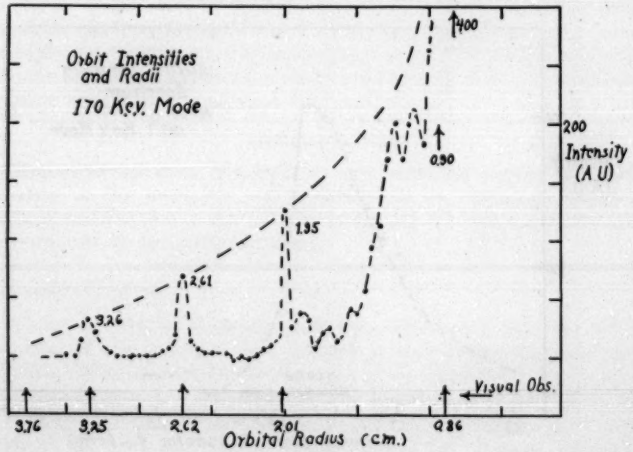

FIG. 10.

is not good. A plausible cause may be found in that there are really two most favorable spots on the resonator throat (intersection of throat circle with median plane) for favorable injection, thus explaining immediately the observed radial difference. This is obliterated by resonator focusing action on subsequent turns.

With the magnetic field at a non-resonant value the electron spectrum obtained showed a maximum Hr of 2800 gauss cm. (475 Kev). Here the electron intensity approached zero and increased inwards toward the resonator to a peak at 822 gauss cm. (56 Kev). This shows that the vast majority of emitted electrons are of low energy and that only a very small fraction of the total emission is trapped into resonance orbits.

Another interesting observation was that of operation in mixed modes which occurred when the magnetic field was about 1500 gauss. The observed data, shown in Table II, may best be interpreted as a

TABLE II.—*Orbits Observed in Mixed Mode Operation.*

Orbit	*a*	*b*	*c*	*d*	*e*	*f*
Radius, cm.	0.93	1.35	1.48	1.73	2.06	2.60
(Hr) g. cm	1400	2196	2214	2600	3097	4007
U. (Kev)	150.4	322.4	326.8	420.8	549.6	794.2
Cor: Theoretical Radius	1.02	1.27	1.52	1.75	1.97	2.60
Identity: Orbit Mode (c_3)	No. 1 1/3	No. 1 1/2 or $r = 1.13$ No. 1 5/12	No. 2 1/3	No. 2 5/12	No. 2 1/2 or No. 3 1/3	No. 3 1/2

competition between 170- and 255-Kev modes by correcting each orbit position to the position it would have in the non-resonant field.

Experiments in which the resonator axis was tilted or displaced in a radial direction showed that the axis must be set closely parallel to the median plane to obtain satisfactory operation and, setting the resonator

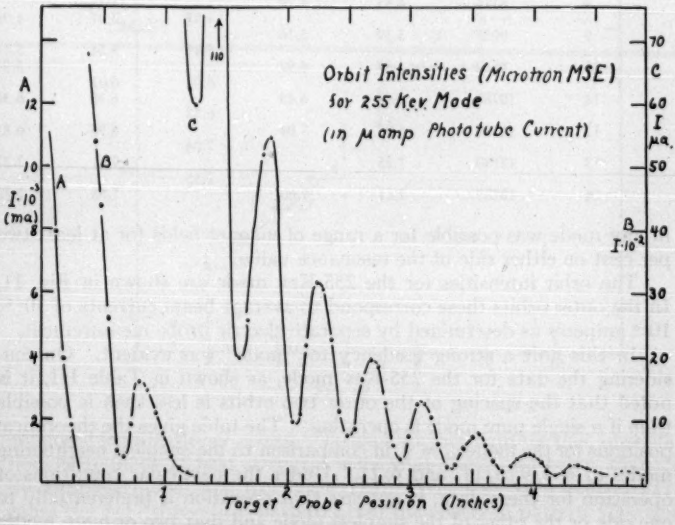

FIG. 11.

too far from the magnet center seriously affects the outer orbits of the pattern, especially as the gain per turn is increased.

Mode and Power Observations on 3.8-Mev Microtron

In its first tests after completion, the 3.8-Mev microtron indicated that it would operate well in the 255-Kev mode, and work in several of the lower modes. Later, using an improved R.F. line tuner, it was possible to observe the stepwise increase in power demand as magnetron voltage was increased and the orbits formed successively in the microtron. Beam intensity and shape could be controlled easily by proper adjustment of R.F. line tuning and magnet current setting. Operation

TABLE III.—*Comparison of Observed and Theoretical 255-Kev Orbits.*

Orbit No.	Hr app. H = 1680	Obs. Radius, cm.	$c_2 = \frac{6}{11}$	Theoretical Radii for Modes $\frac{1}{2}$	$\frac{5}{11}$	$\frac{4}{9}$
1	1848	1.10	1.20	1.14	1.07	1.05
2	2386	1.42			1.65	1.63
			1.87	1.76		
3	3578	2.13		2.33	2.18	2.15
4	4721	2.81	2.48	2.87	2.67	2.63
5	5393	3.21	3.07		3.18	3.11
				3.41		3.59
6	6384	3.80	3.65		3.65	
				3.94		4.05
7	7190	4.28	4.22		4.13	
				4.46	4.60	4.52
8	8114	4.83	4.79			
				4.98	5.07	4.98
9	9055	5.39	5.36			
				5.49	5.54	5.44
10	9862	5.87	5.92			5.91
				6.01	6.01	
11	10786	6.42	6.48		6.48	6.36
				6.53		
12	11726	6.98	7.04		6.95	6.81
				7.04		
13	12180	7.25			7.41	7.27
				7.55		
14	12785	7.61	7.60		7.88	7.72

in any mode was possible for a range of magnet fields for at least two per cent on either side of the resonance value.

The orbit intensities for the 255-Kev mode are shown in Fig. 11. In the outer orbits these correspond to average beam currents of 10^{-8}–10^{-9} amperes as determined by separate electric probe measurement.

In this unit a strong tendency to "mode" was evident. On considering the data for the 255-Kev mode, as shown in Table III, it is noted that the spacing of the outer two orbits is less than is possible than if a single pure mode is operating. The table gives the theoretical positions for the mode $c_2 = \frac{1}{2}$, in comparison to the simplest neighboring modes $c_2 = 4/9$, $5/11$, and $6/11$. Under the particular conditions of operation for these data, it appears that operation is preferentially to one side or the other of the simplest mode and that two or more modes

are operative. Such complex structure may easily be verified by visual exploration with the phosphor target probe.

Data for the 170-Kev mode showed a similar situation. In the outer orbits the 3/10 mode is preferred to the simple 1/3. Although the pattern is predominately in the 3/10 mode, 1/3, 4/11, and 4/9 modes

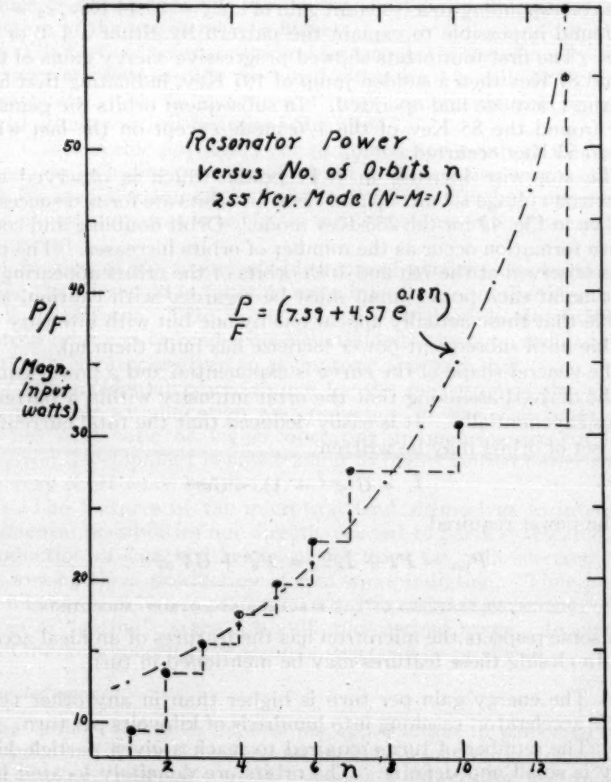

FIG. 12.

are possible in the inner orbits. To shed further light on this, observations were made at field strengths somewhat above and below that for the simplest mode (170 Kev, $c_2 = 1/3$). When set to a field of 1223 gauss the 5/11 theoretical pattern fitted the observed pattern very well

and agreement with the simple 1/3 mode was poor. With the field reduced to 980 gauss the pattern appeared to be in the 1/3 mode for the inner orbits and moved to 3/10 mode for the outer orbits. In this case, however, it happens that the last three orbits can be explained equally well by either mode.

In an orbit pattern observed when the magnetic field was set to 559 gauss corresponding to a resonant gain of $c_2E_0 = 85.14$ Kev, $c_2 = 1/6$, it was found impossible to explain the pattern by either a 1/6 or a 1/5 mode. The first four orbits showed progressive energy gains of 68, 75, 78, and 87 Kev then a sudden jump of 107 Kev, indicating that for one turn the 1/5 mode had operated. In subsequent orbits the gains averaged around the 85 Kev of the 1/6 mode except on the last where a drop to 55 Kev occurred.

The step-wise increase in R.F. power which is observed as the magnetron voltage is built up and electron orbits are formed successively is shown in Fig. 12 for the 255-Kev mode. Orbit doubling and complex pattern formation occur as the number of orbits increases. The peculiarities observed at the 7th and 10th orbits of the orbits appearing three at a time at each power jump must be regarded with caution, as it is possible that these actually appear one by one but with intensity unobservable until subsequent power increase has built them up.

The general shape of the curve is exponential and a power equation may be derived assuming that the orbit intensity within a pattern decreases exponentially. It is easily deduced that the total current for a given set of orbits may be written

$$\bar{I}_s = G(e^{an} - 1) \sim Ge^{an} \tag{19}$$

and the power required

$$P_{tot} = P_0 + \bar{I}_s\bar{V}_s = P_0' + GV_0e^{+an}. \tag{20}$$

DISTINCTIVE FEATURES OF THE ELECTRON CYCLOTRON (MICROTRON)

In some respects the microtron has the features of an ideal accelerator. In closing these features may be mentioned in turn.

(*a*) The energy gain per turn is higher than in any other type of particle accelerator, reaching into hundreds of kilovolts per turn.

(*b*) The number of turns required to reach a given particle kinetic energy is small and definite. The orbits are definitely located in the accelerator and are spaced apart away from the resonator. Electron beam extraction is quite feasible.

(*c*) In view of (*b*) the total time of acceleration of a particle is short—of the order of milli-microseconds. In a high vacuum, particle loss by gas scattering is small. Microtron operation is possible even at pressures greater than 10^{-4} mm. of mercury.

(*d*) Related to (*c*), the radiation is produced in very short bursts, permitting close timing for many experimental studies.

(*e*) A choice of a considerable number of kinetic energies may be had in one and the same accelerator, each energy increment related in some simple ratio to the electron's rest mass energy.

(*f*) The X-radiation output compares favorably with that of other machines of similar size but due to the short pulse the rate of X-ray production at the target may be higher.

(*g*) The first of its constructional features is the use of a very simple static magnetic field readily produced either by d-c. coils or by permanent magnets. Due to shortness of total acceleration time no special effort to obtain vertical focusing is required.

(*h*) Considerable possibility for simplified and compact mechanical construction including possibly of a built-in R.F. power source exists.

(*i*) No special source of electrons is required. Further improvement in field emission sources is today the subject of interesting research and development.

(*j*) In view of (*b*), a controllable beam of particles can be produced allowing the microtron to function as an injector for another large radius particle accelerator. "On orbit" injection is possible since the final microtron orbit can be made co-tangential with the large radius orbit of the large accelerator or given an asymptotic approach to it.

(*k*) The incidental power source for the microtron is also capable of considerable simplification and reduction in size, the latter being somewhat automatic as higher operating frequencies are employed. Magnetron development is now leading to higher pulsed power sources in the very short wave bands (**13**, **14**).

(*l*) The features of the microtron lend themselves to interesting experimental possibilities not directly related to particle research, such as production of bunched beams of electrons for radio-electronic purposes, among them production of mm wave radiation. Time instants when microtron orbits appear successively can be used for sets of time markers or "gating" in the sub milli-microsecond range. In principle a set of markers spaced 10^{-10} seconds apart can be produced.

Acknowledgment

The author wishes to express his appreciation to personnel of the Nucleonics and Radio Divisions for their help in the form of advice or equipment, and especially to Mr. Wesley T. Mayes for his invaluable and conscientious assistance in the construction of apparatus.

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A VERSATILE ELECTRONIC FUNCTION GENERATOR

BY

RAJKO TOMOVICH [1]

SUMMARY

An attempt is made to bring out the characteristic features of electronic circuits destined to cover a wide range of mathematical operations otherwise performed by various individual analog computing elements. Operations in question are essentially nonlinear in character.

Since the device performs the desired operations using a limited number of points, methods are shown how the over-all accuracy can be increased. Use of the proposed technique for different purposes is explained.

In a previous paper (1)[2] an approach to the problems of the nonlinear computing elements in electrical differential analyzers has been proposed, based on a single unit with various mathematical applications. Such a solution is possible in principle, because the majority of the mathematical operations involved may be derived from the expression $\varphi(t) \cdot f[\psi(t)]$.

TABLE I.—*Special Cases of* $\varphi(t) \cdot f[\psi(t)]$.

$\varphi(t)$	$f(x)$	$\psi(t)$	Result
a	$f(x)$	$x = \psi(t) = t$	$a \cdot f(t)$
a	$f(x)$	$x = \psi(t)$	$a \cdot f[\psi(t)]$
a	x^n	$x = \psi(t)$	$a\psi(t)^n$
a	$\sqrt[n]{x}$	$x = \psi(t)$	$a \sqrt[n]{\psi(t)}$
φ	$\frac{1}{x}$	$x = \psi(t)$	$\frac{\varphi(t)}{\psi(t)}$

Table I shows different special cases of this general transformation for definite types of the operators φ, f and ψ.

There are different ways in analog computing to obtain individual operations mentioned in Table I. But little has been said about electronic circuits capable of performing any desired transformation f on two arbitrary voltage waveforms $\varphi(t)$ and $\psi(t)$. The purpose of this paper is to discuss the general properties, accuracy limitations and practical significance of such an electronic device (2), when used in repetitive differential analyzers.

I. DIAGRAM OF THE CIRCUIT

Figure 1 contains the basic components of an electronic system giving at its output a jump function $U_0 = \int \varphi(t) \cdot f[\psi(t)]$. Notation and definition of a jump function are taken from Gardner and Barnes (3).

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

As seen, the system performs the desired operation using only a limited number of points. Errors thus introduced, as well as means to reduce them, will be separately considered.

The necessary functional relation $f(n)$ is set on n potentiometers, P. Reference voltages of the amplitude comparators differ by a convenient constant amount. Under this arrangement, with a linear time base $\psi(t)$, all points identified by the amplitude comparators are equally spaced.

In the general case, with an arbitrary $\psi(t)$, amplitude comparators start a positive pulse whenever $\psi(t)$ crosses a reference voltage regardless of the sign of $\psi'(t)$. These pulses actuate the electronic switches

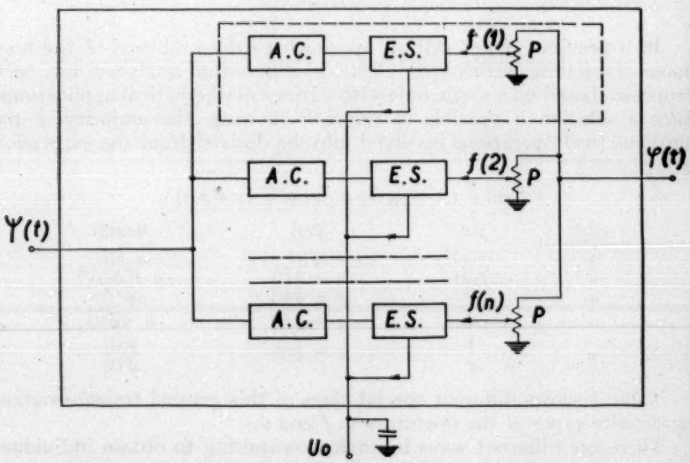

FIG. 1. Electronic circuit with inputs $\varphi(t)$ and $\psi(t)$, and output $U_0 = \int \varphi(t) \cdot f[\psi(t)]$. A.C.—amplitude comparator; E.S.—electronic switch; P—potentiometer.

establishing contact between the corresponding P and output of the device. Assuming the on-time of the switches to be negligible in comparison with the time base of the computer, the expression for U_0 is easily derived.

An important feature of this circuit is that all working points, one being delimited by broken lines on Fig. 1, are identical in design. Such a point comprises an amplitude comparator with positive output pulses, a bi-directional switch and ordinary potentiometer. The whole device may be thus assembled using identical and interchangeable elements, so that the manufacturing and maintenance costs are greatly reduced. In certain cases even all tubes used can be of a single type.

Practical working points vary greatly in design depending on the application of the device, accuracy and time base of the computer. Primary consideration, however, must be given to the reduction of the cost per working point, since the total cost represents a multiple of it. Two types of working points proved useful in various computing circuits.

One type consists of three 6SL7 tubes per point, first for amplitude comparison, second for pulse shaping and third as a bi-directional switch. It has the following characteristics: indeterminacy in amplitude comparison 0.2V, including drift and hysteresis; delay of the whole circuit, defined as difference between time instants when $\psi(t)$ reaches a reference voltage till the sampled value of $\varphi(t)$ appears at output, 30 μsec.; and width of the output pulse 100 μsec.

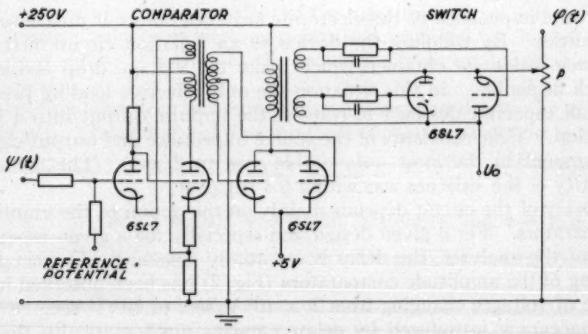

FIG. 2. Diagram of a working point.

The second type, developed to give satisfactory results in various mathematical applications as well as with fast repetition rates (50–60 cps.), consists also of three tubes. A pulse transformer is used to improve appreciably the quality of the comparison process. The following stage takes care that correct pulses activate the switching tube regardless of the sign of $\psi'(t)$. The diagram of the circuit is given in Fig. 2. It should be noted that with this design all special rectifying elements in the second stage are avoided. Inaccuracy in amplitude comparison is 0.1 V, delay 5 μsec., minimum pulse width at output 20 μsec.[3]

Similar circuits, especially with use of miniature tubes and reduction of components, may be conveniently developed for different applications.

II. ACCURACY CONSIDERATIONS

Factors influencing the functioning of the device at points t_n will be discussed first. Means to arrive at better precisions in the whole domain of t are found in the next section.

[3] The design of this circuit was made by T. Aleksich, Assistant in the Laboratory for Applied Mathematics.

In view of the fact that the output U_0 and input voltages $\psi(t)$ and $\varphi(t)$ are linked by the relation $U_0 = \varphi(t) \cdot f[\psi(t)]$, it is clear that the device represents a nonlinear element from the point of view of electric circuit theory. For instance, voltages set on P and appearing at the output, are not affected by any changes in the form of $\psi(t)$, as long as its amplitude surpasses a definite reference potential. Accordingly, the superposition principle, applied in the analysis of linear systems, does not hold here.

Therefore, accuracy considerations met in the design of practical circuits will be divided into following groups: 1. influence of the switching operations on $\varphi(t)$; 2. delay; 3. inaccuracy in amplitude comparison; and 4. width of the output pulse, that is, duration of the on-period of the switches.

Finite impedance of the electronic switches does not create serious difficulties. By coupling the device to an isolation circuit first, impedance ratios are obtained which make the voltage drop across the switch negligible. In this situation the only effective loading presents a small capacitor destined to convert the impulse output into a jump function. Time constants of the source impedance and output capacitor amount in the most unfavorable case to 2 μsec. The measured linearity of the switches was within 0.1 per cent.

Delay of the circuit depends mainly on the action of the amplitude comparators. For a given design and especially for a given repetition rate of the analyzer, the delay is essentially constant. Correct functioning of the amplitude comparators (Fig. 2) has been observed in the range of voltages changing from 2×10^2 V/sec. to 10^7 V/sec.

Inaccuracy introduced by delay $\cdot \tau$ means mathematically that instead of the expected output $\varphi(t_n) f_n[\psi(t_n)]$, one obtains $\varphi(t_n - \tau) \cdot f_n[\psi(t_n - \tau]$. Influences of this shift will be best understood when speaking of various mathematical applications.

Evidently, no amplitude comparator will perform the comparison process always at the same level, no matter how well regulated the reference voltage may be. This inaccuracy, coming mainly from drift of the tubes and hysteresis when comparison is required for either sign of $\psi'(t)$, affects the mathematical operations as follows.

In the ideal case, to every reference voltage, there corresponds one and only one value of $f(n)$. In the practical circuits, however, there is in fact a narrow zone, in our case $a = 0.1$ V, which serves as reference. So $f(n)$ will now correspond to all time coordinates lying inside the interval $\Delta t = \dfrac{a}{\psi'(t)}$. Use of the first differences is justified due to small values of $\Delta\psi(t)$ and Δt. Thus, the existence of a voltage zone determines from the point of view of accuracy a small domain Δt where $f(t) =$ const., $t_n \leq t \leq t_n + \Delta t$.

Influence of the pulse width is considered separately for each operation.

A. Generation $f(t)$

Since here $\varphi(t) = \text{const.}$ and output known, function values are controlled directly after the switches. In other words the ordinates $f(n)$, that is, $f(t_n)$ may be adjusted at output depending chiefly on the accuracy of the measuring equipment.

The same manipulation eliminates the effect of the constant delay τ, because $\psi(t) = kt$ is monotonic. In fact, with this procedure, on P we have $f(t_n + \tau)$ instead of $f(t_n)$. Conclusion is reached that as long as $\tau < \Delta t$, Δt being the smallest interval between two adjacent points, function generation is independent of the repetition rate of $\psi(t) = kt$.

Maximum number of working points is determined by the width of the output pulse. On-time of the switches has no other influence on the accuracy.

Hysteresis also does not enter into consideration due to the special form of $\psi(t)$. Inaccuracy of the amplitude comparison can now be more closely specified. From $\Delta y = f'(t_n) \cdot \Delta t$ and $\Delta t = \dfrac{a}{\psi'(t_n)} = \dfrac{a}{k}$, comes out $\Delta y = f'(t_n) \cdot \dfrac{a}{k}$. For $a = 0.05$ V, without hysteresis, and $k = 1$, a percentage error in Δy with machine unit 100 V equals $\Delta y = f'(t_n) \times 0.05$.

B. Generation $f[\psi(t)]$

Concerning the values of $f(n)$, the situation is not changed because again $\varphi(t) = \text{const.}$ and operator f known. However, the delay τ stays, when an arbitrary form of $\psi(t)$ is assumed. Comparing the given value $\tau = 5$ μsec. with the usual frequencies of the repetitive analyzers (50–60 cps.), we see that the delay is of the order 0.03 per cent.

Hysteresis makes a now equal to 0.1 V. Pulse width has the same meaning as before.

C. Multiplication

Width of the output pulse enters here as an additional factor influencing accuracy, $\varphi(t)$ being arbitrary waveform. Errors introduced by a finite length of the sampling interval are derived from the following reasoning. Evidently they depend on the rate of change of $\varphi(t)$ at that particular instant.

Assuming, for instance, a maximum value $\varphi'(t) = 10$ and an upper error in the product of 1 per cent, length of the sampling interval is $\Delta t = \dfrac{\Delta\varphi(t)}{\varphi'(t)}$, that is, 0.1 per cent. Pulse widths already mentioned fulfill this requirement.

III. APPLICATION

It is the purpose of this section to illustrate the use of the described technique for different mathematical operations. Attention will be given to the methods leading toward a better accuracy in the whole working range.

Such a method, considered satisfactory in conjunction with this technique, represents linear interpolation between two consecutive points t_n. Reasons for this lie in the fact that straight line approximation of a curve—instead of a jump function—reduces greatly the number of working points for a given accuracy. At the same time practically no additional electronic equipment is required besides the one already existing in the differential analyzer.

The case of functions of an independent variable does not offer difficulties for linear interpolation. An electronic integrator is all that is needed. Electronic circuits having the same purpose in the general case $\varphi(t) \cdot f[\psi(t)]$ are also treated.

A. Generation $f(t)$

Straight line approximation of $f(t)$ is best achieved by setting on P, instead of $f(n)$, the first differences $\Delta f = f(n) - f(n-1)$ and integrating the output (Fig. 3). Evaluating the integral $\int_0^t \int \Delta f dt$ we see that it represents a straight line approximation to $f(t)$.

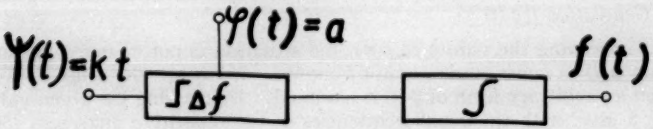

FIG. 3. Generation of $f(t)$.

Naturally the same technique works if all time intervals Δt are not of equal length. Reference voltages in the electronic solution may assume arbitrary values as well as their differences, so that the non-equidistant linear interpolation provides another means to reduce the number of working points.

Using circuit of Fig. 3, for instance, $f(t) = \sin \omega t$, $\omega = 314$, $0 \leq t \leq 0.01$, was generated with fifteen values of t_n, first and last $\Delta t'$ being equal to $\frac{3}{2}\ \Delta t$.

To check the results, a special device for measuring the instanteneous values of periodic voltage waveforms was used (4). All function values were brought within the limits of the mathematical errors due to linear interpolation and inaccuracy of the measuring instrument, that is, $\pm$ 0.5 per cent.

In the study of engineering problems by analog methods, there arises a need for voltage-limiting circuits or other special function generators. Electronic circuits have been developed (5) to obtain the very sharp limiting characteristics necessary in fast analog computers.

It is interesting to note that voltage limiting characteristics and other special nonlinear relations can be obtained precisely with this function generator using only a few points. Even more important is the fact that the simulated relations do not rely on tube characteristics, but come out as results of mathematical operations. They can be simulated to a high degree by ordinary analog computing elements.

Let us take for example the functions represented in Fig. 4(*a*) and 4(*b*). Relation of Fig. 4(*a*) may be adequately expressed as the result

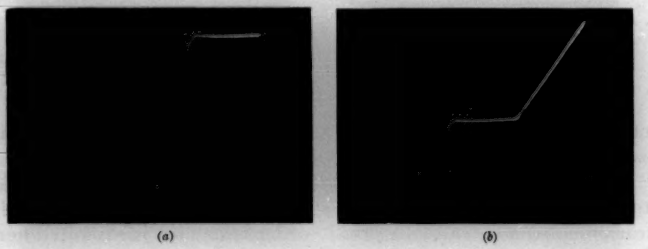

FIG. 4(*a*) and 4(*b*). Examples of special function generation. Sweep duration, 15 μsec.

of integration of the following expression

$$\int_0^t \int f(t)dt + C$$

where

$$C = -a, \int f_1(t) = b[u(t - t_1) - u(t - t_2)], u(t - t_n)$$

is the shifted unit step function, and the function generator is set as follows

$$t = t_1 \qquad f_1(t_1) = b$$
$$t = t_2 \qquad f_1(t_2) = 0.$$

Taking Fig. 4(*b*),

$$C = -a, \int f_2(t) = a[u(t) - u(t - t_1)] + a \cdot u(t - t_2)$$

and

$$t = 0 \qquad f_2(0) = a$$
$$t = t_1 \qquad f_2(t_1) = 0$$
$$t = t_2 \qquad f_2(t_2) = a.$$

Figure 5 shows the block diagram required to carry out the necessary

operations. Only three working points are needed. In view of their characteristics as well as the good frequency response of the electronic integrators, a highly accurate approximation to the mathematical operations involved is possible.

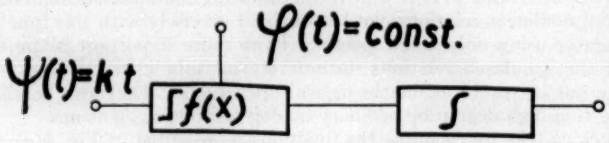

FIG. 5. Block diagram to obtain functions in Fig. 4(*a*) and 4(*b*).

B. Interpolation with $f[\psi(t)]$

Since time intervals Δt depend on an arbitrary $\psi(t)$, means should be provided in this case to evaluate automatically the ratio $\frac{\Delta y}{\Delta t}$. A method for doing this, particularly suited for work in conjunction with this function generator, will be explained.

In the case of the electromechanical function generators, linear interpolation is achieved by aid of special commutator switches and a linear potentiometer (6). In principle the same idea is exploited here, using only electronic components.

Mathematically we need first two jump functions $\int f(n)$ and $\int f(n-1)$, that is their difference, $\int f(n) - \int f(n-1)$. This difference should be applied to a linear potentiometer which will move its tap ac-

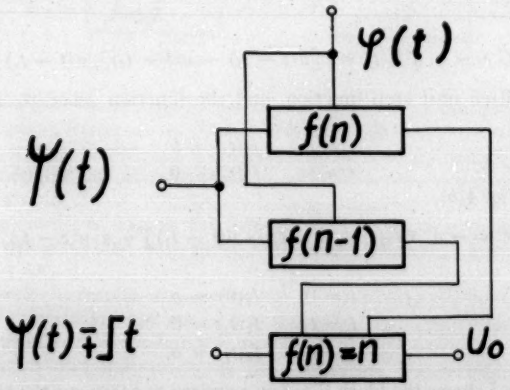

FIG. 6. Disposition of working points for linear interpolation of $f[\psi(t)]$.

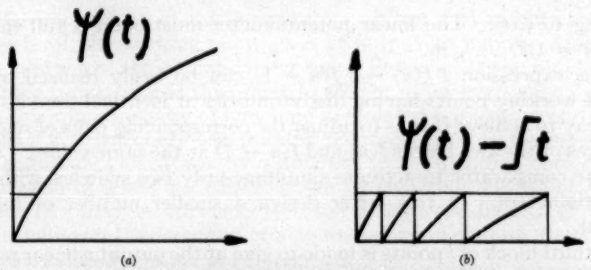

FIG. 7(*a*) and 7(*b*). Functions $\psi(t)$ and $\psi(t) - \int t$.

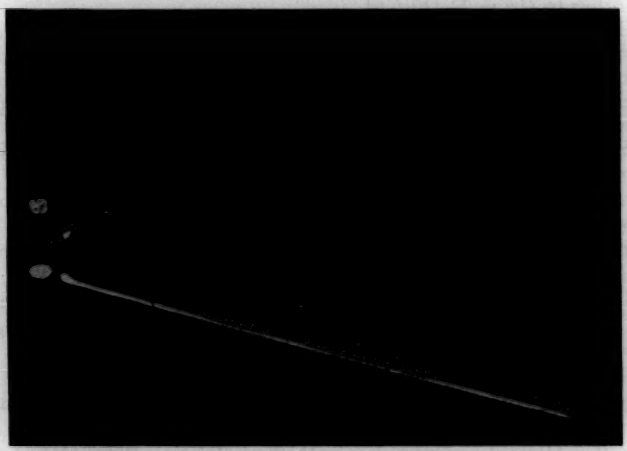

FIG. 8. Special case $\psi(t) = -kt$ and $kt - \int t$. Sweep duration, 15 μsec.

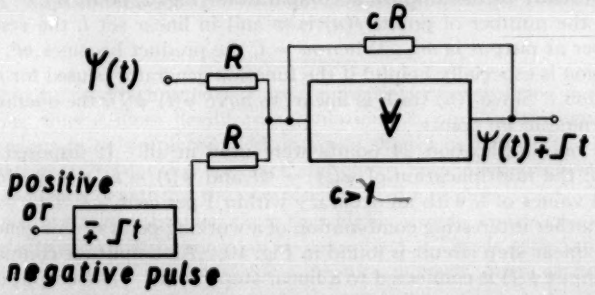

FIG. 9. Obtaining $\psi(t) - \int t$. Timing pulses for the linear step circuit are coming from the amplitude comparators of $f(n)$.

cording to $\psi(t)$. The linear potentiometer must make a full turn for each $n = 1,2, \ldots m$.

The expression $\int f(n) - \int f(n-1)$ can be easily realized by two sets of working points having discontinuities at identical time instants. One way to achieve this, is to adjust the corresponding pairs of reference voltages of the two blocks $f(n)$ and $f(n-1)$ at the same values. Or, to use one comparator to activate simultaneously two switches with their potentiometers. In this latter design a smaller number of tubes is needed.

A third block of l points is made to give at the output a linear relation kt_n. Common ends of its potentiometers are connected one to $f(n)$ and the other to $f(n-1)$ (Fig. 6).

The main problem consists in making $\psi(t)$ cross all reference voltages of the linear block, while in $f(n)$ it moves from one reference level to the next one. In other words, $\psi(t)$ should act as input also to the linear block, but whenever it reaches a reference voltage of $f(n)$, input of the linear block must return to zero.

Subsequently, the input waveform of the third block acting on its amplitude comparators should be of the form $\psi(t) - \int t$, where $\int t$ is a linear jump function whose voltage steps are identical and equal to the amount by which differ the adjacent reference potentials in $f(n)$. Discontinuities in $\int t$ are provoked by the impulses coming from the comparators $f(n)$.

Graphical interpretation of the expression $\psi(t) - \int t$ is given in Fig. 7(a) and 7(b). A special case when $\psi(t) = kt$ is found in Fig. 8. It was generated by the setup represented in Fig. 9.

The block diagram for $\psi(t) - \int t$, as shown in Fig. 9, is simple. To generate $\int t$ any convenient linear step circuit can be used (7). In the general case, the step circuit must provide $\pm\int t$ depending on the sign of $\psi'(t)$. Desired timing pulses with the indication of the sign of $\psi'(t)$ exist already in the amplitude comparators (Fig. 2, point A).

If the number of points $f(n)$ is m and in linear set l, the resulting number at output is ml. When $m = l$, the product becomes m^2. The situation is especially helpful if the function generator is used for multiplication. Since $f(n)$ itself is linear, to have $\varphi(t) \cdot \psi(t)$, the operation is performed in m^2 points.

In one application 24 points were used in all. If disposed as in Fig. 6, the multiplication of $\varphi(t) = kt$ and $\psi(t) = kt$ was performed for 64 values of t, with an accuracy within 1 per cent.

Another interesting combination of a working point of this generator and a linear step circuit is found in Fig. 10. An amplitude comparator with input $\psi(t)$ is connected to a linear step circuit. The output of this step circuit is fed back in such a manner as to change discontinuously

the reference voltage of the comparator. When $\psi(t)$ reaches a preset voltage level, the working point gives at output an impulse depending on the position of the potentiometer tap. Potential at the input of the amplitude comparator is thus reset to zero or other fixed value.

In this way, one may obtain time-modulated impulses according to the signal $\psi(t)$ with only one working point, or, what is the same, perform amplitude comparison at n different levels with one comparator. The number of levels n is easily chosen by changing the tap of P.

This fact may be conveniently used to improve further electronic circuits based on the described principles, having in definite applications a greatly reduced number of components.

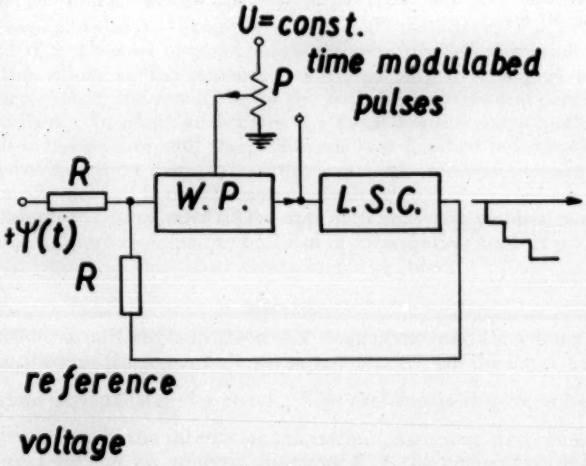

FIG. 10. Amplitude comparison at n levels with one working point. W.P.—working point; L.S.C.—linear step circuit.

IV. CONCLUSION

The proposed technique is of use for a variety of computing purposes, allowing thus a more flexible manipulation of complex mathematical expressions to be simulated. Main stress should be laid on the design improvements to develop compact and cheap basic elements—working points—which constitute the components of the device.

Results already obtained show that highly simple, compact and reliable electronic constructions can be assembled. This is especially true if the technique is employed for a definite purpose, such as function generation or nonlinear mathematical operations.

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V.
19

MIXED BOUNDARY VALUE PROBLEMS IN POTENTIAL THEORY

BY

J. H. HUTH [1]

SUMMARY

Mixed boundary value problems of potential theory are of importance in a diversity of applications. Generally they can best be solved by reduction to a Riemann-Hilbert problem, but this involves certain arbitrary constants. This paper interprets these constants in terms of dipoles.

INTRODUCTION

Perhaps the classical example of a "mixed" boundary value problem is afforded by the case of steady-state temperature determination when given the values of the dependent variable only over a part of the boundary. Over the remainder of the boundary the normal derivative is prescribed. In many such cases the usual Fourier series method of solution is inadequate, and the problem is best handled by reduction to the Riemann-Hilbert case (1).[2] Other examples of importance are to be found in the field of thermal stress analysis (2).

Although the Riemann-Hilbert approach generally yields a result in closed form, there remains the problem of interpreting a set of undetermined constants. This paper treats such a problem.

ANALYSIS

Consider a unit circle in the $z = x + iy$ plane, and let a function ϕ be prescribed on the upper half of the boundary. On the lower half let the normal derivative $\frac{\partial \phi}{\partial n}$ be given. The problem is then to solve Laplace's equation in the interior of the circle. As a first step consider ϕ as the real part of an analytic function Φ of the complex variable z. Now since a pure Neumann problem can be solved by standard techniques, no loss of generality is incurred by assuming $\frac{\partial \phi}{\partial n} = 0$ along the lower portion of the boundary. It is then permissible to take the imaginary part of Φ equal to zero thereon.

Consequently Φ can be extended to the whole plane by means of a reflection. That is, let Φ be defined for $|z| > 1$ in terms of Φ inside the unit circle by means of the formula:

$$\Phi(z) = \overline{\Phi\left(\frac{1}{\bar{z}}\right)}. \tag{1}$$

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

Then Φ will be analytic in the entire plane with the exception of a semi-circular cut corresponding to the upper half of the original boundary. If $f(\theta)$ represents the prescribed values of ϕ along the arc, then Φ must satisfy a cut condition

$$\Phi^+ + \Phi^- = 2f(\theta) \tag{2}$$

where the $+$ and $-$ superscripts indicate values obtained by approaching the cut from opposite sides. In this form the problem is that of Reimann-Hilbert. It should be noted however that this formulation as stated does not have a unique solution, and not every solution will necessarily solve the original real variable problem in ϕ.

Let X be a solution of the homogeneous problem:

$$X^+ + X^- = 0. \tag{3}$$

X can then be taken in the form

$$X = \frac{\sum_{n=0}^{\infty} a_n z^n}{\sqrt{z^2 - 1}} \tag{4}$$

provided that the branch line for the square root is taken along the cut. The a_n's are complex constants and their interpretation is precisely the object of this paper. Before proceeding in this direction, the solution of the original Riemann-Hilbert problem will be finished for the sake of completeness.

Equation 2 can now be written in the form:

$$\left(\frac{\Phi}{X}\right)^+ - \left(\frac{\Phi}{X}\right)^- = 2\,\frac{f}{X^+}. \tag{5}$$

Then $\left(\frac{\Phi}{X}\right)$ is expressible in integral form as follows by virtue of Cauchy's formula for arcs.

$$\left(\frac{\Phi}{X}\right) = \frac{1}{2\pi i}\int_L \frac{2f(t)}{X^+(t)\,(t - z)}\,dt. \tag{6}$$

Here t is a running variable along the arc L. Of course the complete solution of the Riemann-Hilbert problem can be written as:

$$\Phi = \frac{\sqrt{z^2 - 1}}{2\pi i}\int_L \frac{2f(t)}{\sqrt{t^2 - 1}\,(t - z)}\,dt + \frac{\sum_{n=0}^{\infty} a_n z^n}{\sqrt{z^2 - 1}}. \tag{7}$$

The line integral can be easily related to a contour integral around the cut and then evaluated by residues.

To interpret and obtain the restrictions on the a_n's it first is necessary to recall that the objective is to have X provide a non-trivial solution of

the original problem in real variables with $\phi = 0$ on the upper half, and $\frac{\partial \phi}{\partial n} = 0$ on the lower half of the unit circle boundary. But this problem is capable of treatment by an entirely different approach. It is well known that Möbius' transformation (3) will map a half circle with base horizontal in the w plane into a full circle in the z plane.

Specifically, let the problem in the w plane be to find a harmonic function vanishing on the circular part of the boundary of a half-circle, and with vanishing normal derivative along the base. Then Möbius' transformation will result in exactly the problem to be solved by X.

It is immediately seen that a complex dipole (and only a dipole) placed at either corner of the half-circle, together with an additive constant potential, will satisfy the boundary conditions (4). In other words the homogeneous problem in the w plane can be solved by the following expression:

$$X(w) = A\left[\frac{1}{w+1} - \frac{1}{2}\right] + B\left[\frac{1}{w-1} + \frac{1}{2}\right]. \tag{8}$$

Here A and B are arbitrary complex strengths of the left and right dipoles, respectively. Written out, Möbius' transformation reads:

$$z = -i\frac{w^2 + 1 + 2iw}{w^2 + 1 - 2iw}. \tag{9}$$

Combining Eqs. 8 and 9 yields the final result:

$$X(z) = -\frac{1}{2\sqrt{2}} \times \left\{ \frac{\{B(i-1) - A(i+1)\} + \{A(1+i) + (i-1)B\}z}{\sqrt{z^2 - 1}} \right\}. \tag{10}$$

Comparing Eq. 10 with Eq. 4 yields the necessary conditions on the a_n's in order for the Riemann-Hilbert solution to also solve the real variable problem. All the a_n's must vanish except a_0 and a_1. Hence $X(z)$ must remain finite at infinity. Further, Eq. 10 establishes two relations between the real and imaginary parts of a_0 and a_1, so that only two arbitrary constants remain. Of course the same result could have been obtained directly from (7) by referring back to the original boundary conditions on ϕ. However the above derivation gives the physical significance of dipoles in the w plane.

CONCLUSIONS

It has been shown that specifying ϕ and $\frac{\partial \phi}{\partial n}$ along the two segments of a boundary is, in general, not sufficient to completely determine a

harmonic ϕ in the interior of the region. In addition, it is necessary to give information concerning singularities at the junctures of the segments. These singularities have been interpreted in terms of two arbitrary dipoles. These concepts can, of course, be extended to more general cases involving a larger number of boundary segments.

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SOME FORMULAS FOR LARGE DEFLECTIONS OF BEAM COLUMNS

BY

B. SAELMAN [1]

INTRODUCTION

In the analysis of a structural member, consideration should always be given as to whether the deflections involved are within the applicability of the formulas that are used. Formulas derived under the assumption that deflections are small and high order infinitesimals can be neglected can lead to large unconservative errors when applied to conditions involving large deflections. Bisshopp and Drucker, Barten and Conway (1, 2, 3)[2] provide large deflection solutions for the cantilever with a concentrated end load and the simply supported beam with a central concentrated load. Timoshenko (6) discusses large deflections of a column. This paper considers the cantilever beam column, the simply supported centrally loaded beam column, and the cantilever loaded by its own weight or by an ice load.

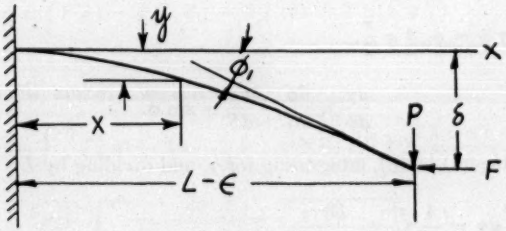

FIG. 1.

We have by fundamental theory, for the case illustrated in Fig. 1,

$$\frac{d\phi}{dS} = \frac{M}{D} = \frac{P}{D}(L - \epsilon - X) + \frac{F}{D}(\delta - y),\ D = \frac{EI}{1 - \mu^2} \qquad (1)$$

$$\frac{d^2\phi}{dS^2} = \frac{P}{D}\left(-\frac{dx}{dS}\right) + \frac{F}{D}\left(-\frac{dy}{dS}\right) = -\frac{P}{D}\cos\phi - \frac{F}{D}\sin\phi. \qquad (2)$$

With the extreme condition,

$$\frac{d\phi}{dS} = 0 \text{ when } \phi = \phi_1$$

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

and integrating, (2)

$$\frac{d\phi}{dS} = \frac{\sqrt{2}}{\sqrt{D}} (F \cos \phi - P \sin \phi + P \sin \phi_1 - F \cos \phi_1)^{\frac{1}{2}} \tag{3}$$

$$\frac{d\phi}{dS} = \sqrt{2} \sqrt[4]{\frac{F^2 + P^2}{D^2}} [\sin (\phi_1 - \psi_0) - \sin (\phi - \psi_0)]^{\frac{1}{2}} \tag{4}$$

where

$$\sin \psi_0 = \frac{F}{\sqrt{F^2 + P^2}}, \cos \psi_0 = \frac{P}{\sqrt{F^2 + P^2}}. \tag{5}$$

Notice $F = 0$ when $\psi_0 = 0$.
Letting

$$1 + \sin (\phi - \psi_0) = [1 + \sin (\phi_1 - \psi_0)] \sin^2\theta = 2 k^2 \sin^2 \theta \tag{6}$$

then, when $\phi = 0$,

$$\sin \theta_1 = \sqrt{\frac{1 - \sin \psi_0}{1 + \sin (\phi_1 - \psi_0)}} = \sqrt{\frac{1 - \sin \psi_0}{2 k^2}} \tag{7}$$

and, when $\phi = \phi_1, \theta = \frac{\pi}{2}$,

$$\frac{dy}{d\phi} \times \frac{d\phi}{dS} = \frac{dy}{dS} = \sin \phi. \tag{8}$$

Substituting (4) in (8), integrating for y, and dividing by L,

$$\frac{\delta}{L} = \frac{1}{L} \int_0^{\delta} dy = \frac{1}{\sqrt{2}} \sqrt[4]{\frac{D^2}{(F^2 + P^2) L^4}} \times \int_0^{\phi_1} \frac{\sin \phi \, d\phi}{[\sin (\phi_1 - \psi_0) - \sin (\phi - \psi_0)]^{\frac{1}{2}}} \tag{9}[3]$$

$$\phi = \sin^{-1} (2 k^2 \sin^2 \theta - 1) + \psi_0 = \eta + \psi_0 \tag{10}$$

$$\sin \phi = \sin (\eta + \psi_0) = \sin \eta \cos \psi_0 + \sin \psi_0 \cos \eta$$
$$= (2 k^2 \sin^2 \theta - 1) \cos \psi_0 + \sin \psi_0 (2 k \sin \theta \sqrt{1 - k^2 \sin^2 \theta}) \tag{11}$$

where $\sin \eta = 2 k^2 \sin^2 \theta - 1$, and

$$\cos \eta = 2 k \sin \theta \sqrt{1 - k^2 \sin^2 \theta}. \tag{12}$$

[3] The assumption is made that no deflections of the mid-plane of the beam occur due to tension or compression.

Differentiating (6) and solving for $d\phi$

$$d\phi = \frac{4k^2 \sin\theta \cos d\theta}{\cos(\phi - \psi_0)} = \frac{2\,k\cos\theta\,d\theta}{\sqrt{1 - k^2\sin^2\theta}}. \tag{13}$$

Using (6), (11) and (13) in (9)

$$\frac{\delta}{L} = \frac{1}{\sigma}\int_{\theta_1}^{\pi/2} \frac{[\cos\psi_0\,(2\,k^2\sin^2\theta - 1) + \sin\psi_0\,(2\,k\sin\theta\,\sqrt{1 - k^2\sin^2\theta})]d\theta}{(1 - k^2\sin^2\theta)^{\frac{1}{2}}}$$

$$= \frac{\cos\psi_0}{\sigma}\left[\int_{\theta_1}^{\pi/2} \frac{d\theta}{(1 - k^2\sin^2\theta)^{\frac{1}{2}}} - 2\int_{\theta_1}^{\pi/2}(1 - k^2\sin^2\theta)^{\frac{1}{2}}\,d\theta\right] + \frac{2\,k\sin\psi_0\cos\theta_1}{\sigma}, \tag{14}$$

where

$$\sigma = \sqrt[4]{\frac{(F^2 + P^2)L^4}{D^2}}. \tag{15}$$

Using the notation of Janke and Emde (9), (14) can be expressed in terms of a combination of elliptic integrals and trigonometric functions,

$$\frac{\delta}{L} = \frac{1}{\sigma}\{\cos\psi_0\,[F(k) - F(k, \theta_1) - 2\,E(k) + 2\,E(k, \theta_1)] + 2\,k\sin\psi_0\cos\theta_1\} \tag{16}$$

From (4), (6), and (7), we can write

$$\sqrt{2}\,\sigma = \int_{\theta_1}^{\pi/2} [2\,k^2 - 1 - (2\,k^2\sin^2\theta - 1)]^{-\frac{1}{2}} \frac{2\,k\cos\theta\,d\theta}{\sqrt{1 - k^2\sin^2\theta}} \tag{17}$$

$$\sigma = \int_{\theta_1}^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2\sin^2\theta}} = F(k) - F(k, \theta_1). \tag{18}$$

Using (18) in (16),

$$\frac{\delta}{L} = \cos\psi_0\left[1 - 2\,\frac{E(k) - E(k, \theta_1)}{F(k) - F(k, \theta_1)}\right] + \frac{2\,k\sin\psi_0\cos\theta_1}{F(k) - F(k, \theta_1)}. \tag{19}$$

Now, from (1) and (4),

$$\left.\frac{d\phi}{dS}\right|_{\phi=0} = \frac{M_{\max}}{D} = \frac{P(L - \epsilon)}{D} + \frac{F\delta}{D}$$

$$= \sqrt{2}\sqrt[4]{\frac{F^2 + P^2}{D^2}}\,[\sin(\phi_1 - \psi_0) + \sin\psi_0]^{\frac{1}{2}}. \tag{20}$$

From (6)

$$\sin(\phi_1 - \psi_0) = 2\,k^2 - 1. \tag{21}$$

Dividing (20) by $\sqrt{F^2 + P^2}$ and using (21) in (20)

$$\frac{P(1-\epsilon)}{D\sqrt{F^2+P^2}} + \frac{F\delta}{D\sqrt{F^2+P^2}} = \frac{\sqrt{2}}{\sqrt{F^2+P^2}} \times \sqrt[4]{\frac{F^2+P^2}{D^2}}\,(2k^2 - 1 + \sin\psi_0)^{\frac{1}{2}}. \quad (22)$$

Using (15) in (22), dividing by L, and simplifying,

$$\frac{(L-\epsilon)}{L} = \frac{\sqrt{2}}{\sigma\cos\psi_0}\,(2k^2 - 1 + \sin\psi_0)^{\frac{1}{2}} - \frac{\delta}{L}\frac{\sin\psi_0}{\cos\psi_0}. \quad (23)$$

$\frac{\delta}{L}$ in Eq. 23 can be determined from (19).

With δ and $L - \epsilon$ evaluated by (19) and (23),

$$M_{max} = P(L-\epsilon) + F\delta.$$

Curves can be plotted from (19) and (23) which will find δ and $(L - \epsilon)$ when P, F, D and L are given.

It is noteworthy that although a value of k would be assumed in plotting deflection *versus* loads, it can be shown that k is actually a function of the given parameters, F, P, and L and D. This is seen from Eqs. 5, 15, 7 and 18. Equations 5 and 7 give θ_1 in terms of k, and (15) and (18) also express θ_1 in terms of k. Since these pairs are independent, k and θ_1 are uniquely determined in terms of F, P, L, and D. The small deflection theory gives (7)

$$\delta = -\frac{P}{F}\left[\sqrt{\frac{EI}{F}}\tan\frac{L-\epsilon}{\sqrt{\frac{EI}{F}}} - (1-\epsilon)\right].$$

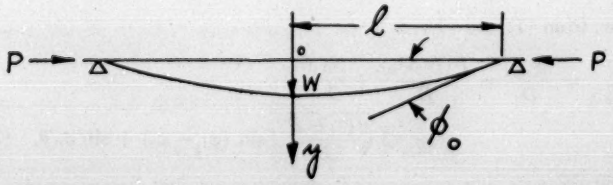

FIG. 2.

For the case shown in Fig. 2,

$$\delta = \frac{\sqrt{2}\sin\alpha}{\sqrt{c^2+c_1^2}}\sqrt{\sin(\phi_0+\alpha)-\sin\alpha}+\frac{\cos\alpha}{\sqrt{c^2+c_1^2}} \times [F(k,\theta_1)-F(k)+2E(k)-2E(k,\theta_1)] \quad (24)$$

where

$$\sin(\phi_0+\alpha) = \frac{1}{2\sqrt{c^2+c_1^2}}(c\delta+c_1 l)^2+\sin\alpha \quad (25)$$

divide EI by $(1-\mu^2)$

$$C = \frac{P}{EI}, \qquad C_1 = \frac{W}{2EI} \quad (26)$$

$$k^2 = \frac{1+\sin(\alpha+\phi_0)}{2}$$

$$\sin\alpha = \frac{C}{\sqrt{c^2+c_1^2}} \qquad \sin\theta_1 = \sqrt{\frac{1+\sin\alpha}{1+\sin(\alpha+\phi_0)}}. \quad (27)$$

Equations 24 and 25 can be solved for $\sin(\phi_0+\alpha)$ and δ. The small deflection theory for this case yields (8)

$$\delta = -\frac{1}{2}\frac{W}{P}\sqrt{\frac{EI}{P}}\left[\tan\frac{1}{2}\frac{l}{\sqrt{\frac{EI}{P}}}-\frac{1}{2}\frac{l}{\sqrt{\frac{EI}{P}}}\right]. \quad (28)$$

For the case of a cantilever loaded by its own weight or by an ice load (see Fig. 3), let

$$\frac{EI}{1-\mu^2} = D.$$

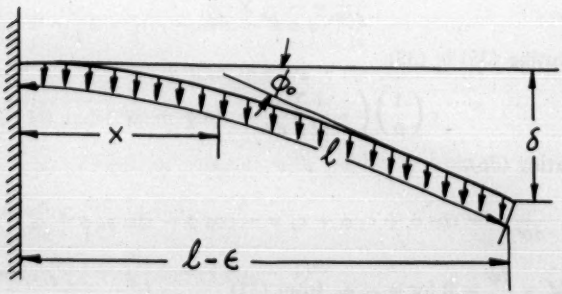

FIG. 3.

Then

$$D\frac{d^2\phi}{dS^2} = w[x - (l - \epsilon)] \tag{29}$$

$$D\frac{d^3\phi}{dS^3} = w\frac{dx}{dS} = w\cos\phi. \tag{30}$$

Now

$$\frac{d^3\phi}{dS^3} = \frac{d}{dS}\left(\frac{d^2\phi}{dS^2}\right) = \frac{d}{d\phi}\left[\frac{d\phi}{dS}\frac{d^2\phi}{dS^2}\right]. \tag{31}$$

Using (31) in (30)

$$\frac{D}{w}d\left[\frac{d\phi}{dS}\left(\frac{d^2\phi}{dS^2}\right)\right] = \cos\phi\, d\phi. \tag{32}$$

Integrating (32),

$$C\frac{d\phi}{dS}\left(\frac{d^2\phi}{dS^2}\right) = \sin\phi + c_1, \qquad \text{where } C = \frac{D}{w}. \tag{33}$$

Since $M = 0$ when $\phi = \phi_0$,

$$c_1 = -\sin\phi_0 \tag{34}$$

Letting

$$p = \frac{dS}{d\phi}, \qquad \frac{1}{p} = \frac{d\phi}{dS}, \qquad \frac{d\left(\frac{1}{p}\right)}{dS} = \frac{d^2\phi}{dS^2}$$

and

$$\frac{d\left(\frac{1}{p}\right)}{dS} = \frac{d\left(\frac{1}{p}\right)}{d\phi} \times \frac{d\phi}{dS} = \frac{d\left(\frac{1}{p}\right)}{d\phi} \times \frac{1}{p} = -\frac{1}{p^2}\frac{dp}{d\phi} \times \frac{1}{p},$$

or

$$\frac{d^2\phi}{dS^2} = -\frac{1}{p^3} \times \frac{dp}{d\phi}. \tag{35}$$

Substituting (35) in (33)

$$c\left(\frac{1}{p}\right)\left(-\frac{1}{p^3}\right)\frac{dp}{d\phi} = \sin\phi + c_1. \tag{36}$$

Integrating (36),

$$\frac{c}{3p^3} = -\cos\phi + c_1\phi + c_2 = -\cos\phi - \sin\phi_0\,\phi + c_2. \tag{37}$$

Since $M = \dfrac{d\phi}{dS} = 0$ for $\phi = \phi_0$, from (37)

$$\frac{d\phi}{dS} = \frac{1}{p} = \sqrt[3]{\frac{3}{c}}\sqrt[3]{c_2 - \phi_0 \sin \phi_0 - \cos \phi_0} = 0 \tag{38}$$

and

$$c_2 = \phi_0 \sin \phi_0 + \cos \phi_0.$$

Integrating (37)

$$\sqrt[3]{\frac{c}{3}}\, l = \int_0^{\phi_0} \frac{d\phi}{\sqrt[3]{c_2 + c_1 \phi - \cos \phi}}. \tag{39}$$

Now, since

$$\frac{dy}{dS} = \frac{dy}{d\phi} \times \frac{d\phi}{dS} = \sin \phi \tag{40}$$

using (38) in (40), we have

$$\frac{dy}{d\phi}\left[\sqrt[3]{\frac{c}{3}}\sqrt[3]{c_2 + c_1 \phi - \cos \phi}\right] = \sin \phi. \tag{41}$$

Integrating (41),

$$\sqrt[3]{\frac{c}{3}}\, \delta = \int_0^{\phi_0} (c_2 + c_1 \phi - \cos \phi)^{-1} \sin \phi \, d\phi. \tag{42}$$

Multiplying Eq. 39 by c_1 and adding it to Eq. 42, we have

$$\int_0^l c_1 \sqrt[3]{\frac{c}{3}}\, dS + \sqrt[3]{\frac{c}{3}}\, \delta = \int_0^{\phi_0} \frac{(c_1 + \sin \phi)}{\sqrt[3]{c_2 + c_1 \phi - \cos \phi}}\, d\phi. \tag{43}$$

The numerator of the integral of (43) is an exact differential of the expression under the radical; hence,

$$c_1 \sqrt[3]{\frac{c}{3}}\, l + \sqrt[3]{\frac{c}{3}}\, \delta = \tfrac{3}{2}(c_2 + c_1 \phi - \cos \phi)^{2/3}\Big|_0^{\phi_0}$$
$$= \tfrac{3}{2}[(c_2 + c_1 \phi_0 - \cos \phi_0)^{2/3} - (c_2 - 1)^{2/3}]. \tag{44}$$

From (34) and (38)

$$c_2 + c_1 \phi_0 - \cos \phi_0 = 0, \quad \text{and } c_1 = -\sin \phi_0. \tag{45}$$

Substituting (45) in (44)

$$-\sin \phi_0 \sqrt[3]{\frac{c}{3}}\, l + \sqrt[3]{\frac{c}{3}}\, \delta = -\tfrac{3}{2}(c_2 - 1)^{2/3}$$
$$= -\tfrac{3}{2}[\cos \phi_0 + \phi_0 \sin \phi_0 - 1]^{2/3}. \tag{46}$$

Solving for δ in Eq. 45,

$$\delta = \sqrt[3]{\frac{3}{c}}\left[\sin\phi_0\sqrt[3]{\frac{c}{3}}\,l - \tfrac{2}{3}(\cos\phi_0 + \phi_0\sin\phi_0 - 1)^{2/3}\right]. \qquad (47)$$

From Eq. 39 one can solve ϕ_0 as a function of l and c; then substituting this expression in Eq. 47, δ is expressed as a function of c and l.

CONCLUSION

The problems of large deflections of the simple cantilever and the simply supported beam with a single concentrated load have been solved. The logical sequence to these solutions are presented in this paper.

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V.
19

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

CONVEYER BELT SORTER

An improved machine for sorting physical objects into a large number of categories has recently been developed by the National Bureau of Standards. Designed by J. Rabinow of the NBS electromechanical ordnance laboratory at the request of the Bureau of Census, the device was built to sort punched cards at the rate of 420 cards per minute. The principle, however, is applicable to sorting such other objects as mail, electrical and mechanical components, and even farm produce as well as checks, invoices, and other papers. Any items that can be separated into a number of subdivisions can be handled by an electromechanical system similar to the NBS sorter.

There are several fundamental approaches to the problem of sorting objects into many classifications. One approach commonly used is to place the objects to be sorted on a conveyer and then to examine them, one by one, as they pass the various sorting stations. Another method, used particularly in sorting punched cards, is to examine all the cards individually at a single reading station and then to direct them immediately into various tracks. A third method, used in the new NBS sorter, is to examine the objects at one station, feed them into a conveyer, and simultaneously put a mechanical address number into each section of the conveyer belt so that the object and its address number travel together. When the object reaches the correct address, or location, a simple recognition device is excited, and the object is removed from the conveyer.

The first two methods have the disadvantages of being far too cumbersome and expensive, or of being limited to a relatively small number of sortings. The third method, however, will separate items into a large number of categories with only one examining device and comparatively simple terminal equipment.

The machine developed at the National Bureau of Standards consists of five major components: (1) a sensing unit that reads the data-bearing cards and decides where they should go; (2) an addressing device that loads the conveyer with the cards and their corresponding address numbers; (3) a conveyer belt that carries both the card and its address number; (4) a series of recognition devices, actuated by the information-bearing mechanism of the conveyer belt, which operate trip mechanisms so that the cards are released from the conveyer; and (5) a series of receptacles, or gates to other devices, into which the material borne by the conveyer is sorted.

* Communicated by the Director.

The NBS machine contains a track on which the cards are pushed by fingers extending down from the conveyer belt directly above. The conveyer belt conveys information in coded form, and the cards are kept in the correct physical location with respect to the address number by the pusher fingers. The conveyer belt is made from two parallel chains whose links are four inches on center and are connected by a crossbar at each card position. Each crossbar carries two Nylon wheels on which the chains travel and eight Nylon cams that act as the mechanical binary address number.

Each of the cams can be set to one of two positions (corresponding to the binary "zero" or "one") with the point either down or up. They arrive at the starting end of the conveyer in the clear or "zero" position. If it is necessary to tip the cam upward so that it becomes the equivalent of the binary "one," a pawl is moved into its path. Eight cams, each with two possible positions, provide 2^8 or 256 possible binary addresses. Since in general there are 2^n different addresses possible with n cams, it can be seen that the number of categories into which a quantity of items may be sorted is limited only by the number of cams carried by the crossbars connecting the two chains.

In the sorting operation, the cards are first dealt from the bottom of the stack and pass under a conventional 80-column card reader. When the information on the card is decoded, an electrical device decides into which pocket the card is to go, and the cams are set up with the correct address number. The card is then pushed onto the track. As the cams move down the conveyer with their associated cards, they pass under a series of switches, one set at each pocket. A cam in the upward position will actuate a switch; one in the downward position will not. By properly wiring a set of eight switches, any particular 8-digit binary address number can be recognized, exciting a mechanism that will deflect the card from the track into a pocket.

While conveyer belt sorters of this general type are not new, the NBS machine has several definite advantages. It is not a synchronous machine; that is, the card spacing in the conveyer does not have any relation to the spacing of the pockets. This leads to a saving in the power supply inasmuch as the cards can not all be dropped simultaneously. However, if the pockets were to have the same spacing as the cards on the conveyer belt, then the cards could be dropped simultaneously although this would not normally occur. Large sections of the belt may be added, as well as groups of pockets in any desired spacing, but the recognizing element in all cases must be spaced a constant distance from its associated pocket. It is also a simple matter to provide more than one pocket for one category; when one pocket fills, subsequent cards can be deposited in a second pocket. Moreover, it is relatively easy to provide a counter associated with each pocket.

If a machine similar to the NBS model were to be designed for sorting mail, it is expected that for some time to come human operators would

have to read the addresses on the letters. By means of suitable keyboards, the operators would set up the conveyer belt as the mail is fed into the conveyer. As the speed of the conveyer would normally be much higher than could possibly be matched by one operator, several people would be placed along the belt to feed it. Each operator would set up a series of pawls corresponding to the address on his letter and would then press a transfer key. The conveyer belt cams would be set up, and the letter (or package) would be inserted into the conveyer at the same time. It would be quite simple to provide an additional cam in each set to indicate that a particular space is filled; this would prevent a letter from being transferred until an empty section had arrived at the feeding station.

MINIATURE ELECTROSTATIC GENERATOR

A miniature electrostatic source of high voltage for use with radiation survey instruments has been developed by S. R. Gilford, S. Saito, and J. L. Herson of the National Bureau of Standards. The device is an adaptation of the work of Holtz and Wommelsdorf[1] on influence-type generators of conventional size, but it uses modern printed circuit and miniaturization techniques. Operation of this type of generator depends on the ability of one charged body to induce a charge on another body close by. The device is one result of a program sponsored by the Navy Bureau of Ships for the investigation and application of techniques adaptable to low-cost mass production of radiation survey instruments.

Many instruments for the detection and measurement of nuclear radiation require high-voltage sources of low current to supply polarization potentials for ionization chambers. Conventionally this voltage is obtained from miniature high-voltage batteries used directly or in combination with capacitors charged in parallel and then connected in series, from a vacuum-tube oscillator in combination with a high-ratio transformer and rectifier, or from a vibrator power supply. The NBS generator that is designed to replace these is simpler to fabricate, is potentially less expensive, has fewer components, and avoids the use of special batteries that may be difficult to obtain.

The alpha survey meter using the air proportional counter provides an opportunity for application of this electrostatic source. The probe requires 2000 volts at a current of 10^{-12} to 10^{-14} amp., which can be obtained conveniently from a storage capacitor recharged periodically by the new generator.

The generator consists basically of a stator of two field plate conductors, and a rotor with a number of pairs of conducting sectors. Printed circuit techniques are used to apply the conducting areas to the flat insulating plates of the rotor and stator. Several sets of brushes

[1] U. S. Patent No. 1,071,196, August 26, 1913.

transfer electrical charge between the components of the system and the storage capacitor. The attached driving system enables the rotor to be driven at speeds as high as 6000 rpm. by a reciprocating drive mechanism operated by an external lever. Although simpler drive systems were devised, the reciprocating type was chosen for its convenience, small size, and the ease with which it can be adapted to a hermetically sealed generator.

One problem in the development of the NBS generator was the devising of a method to establish a unique polarity of output voltage. The randomness of the initial charge distribution which determines the output polarity can be overcome in several ways. One method relies on the use of a dielectric material which retains charge for a long period of time. Another scheme utilizes the triboelectric potential of different brush materials and dielectrics to establish the initial charge distribution. A third method, used in the NBS instrument, requires the use of a small external bias voltage to precharge the generator. The biasing method was adopted because it is the most reliable and because the polarizing potential is already available in the associated electronic equipment.

Experiments indicate that size places no particular limitation on the successful performance of the printed circuit generators. Practical units as small as $1\frac{5}{8}$ in. in diameter have been constructed. As the size decreases, however, the output current decreases correspondingly; the disadvantage here is that a longer time is required to charge the storage capacitor. The spacing of the elements having maximum potential stress determines the output voltage.

Experience with the NBS electrostatic generator shows that a 0.02-microfarad capacitor can be charged to two kilovolts in about 15 sec. In use in the radiation survey instrument, the capacitor needs to be recharged only occasionally to make up a leakage loss of about 100 volts. This can be replenished by the generator in a single operation of the lever on the driving mechanism.

DETERMINATION OF ALDEHYDE IN GLUCOSE

Current studies at the National Bureau of Standards suggest a promising method of attack in chemical investigations of the degradation of cellulose. In the NBS work, procedures using acid sodium chlorite for the quantitative determination of aldehyde groups in glucose—one of the principal building blocks in the cellulose molecule—have been developed by H. F. Launer,[1] W. K. Wilson, and J. H. Flynn of the Bureau staff. The presence of increasing amounts of aldehyde groups in carbohydrate polymers is related to molecular degradation.

The cellulose in wood, paper, textiles, and other common materials is a high polymer built up from carbohydrate monomer units, chiefly glucose. Other aldose sugars—such as xylose, arabinose, and galactose—as well as modified sugar units, may also exist in cellulose.

[1] Now at Western Regional Research Laboratory, U.S.D.A., Albany, Calif.

Aldose sugars, as suggested by the name, contain an aldehyde group. In carbohydrate polymers, such as cellulose, this aldehyde group is initially tied up in the molecular chain with the exception of the unit at one end of the chain. But when the polymeric molecule has been modified by the action of light, heat, or chemical reagents, it may contain other free aldehyde groups in addition to that which occurs at the end of the chain. Methods for determination of aldehyde in aldose sugars are thus of considerable importance for research into mechanisms of cellulose deterioration.

Commonly accepted methods for the determination of aldehyde in cellulose employ an alkaline medium. Although alkaline methods are often useful, some modified celluloses are quite sensitive to alkali and may be hydrolyzed or otherwise degraded by it. This results in an ambiguous result for aldehyde content. Acid solutions of sodium chlorite, on the other hand, do not attack functional groups in cellulose or sugars with the exception of the aldehyde group, which is quantitatively oxidized to carboxyl. This fact suggested that aldehyde in cellulose could be determined by means of oxidation with acid chlorite. In connection with a general program on the degradation of cellulose, NBS has therefore undertaken an investigation of the quantitative oxidation of aldose sugars by means of acid solutions of sodium chlorite.

A difficulty with the sodium chlorite method is that solutions of chlorite salts are unstable in acid medium. As chlorite reacts rather slowly with aldehyde in hexose sugars, appreciable decomposition of chlorite occurs during the reaction. In preliminary work at NBS with hexose sugars, particularly glucose, this difficulty has been overcome in two ways: (1) by use of a calibration curve relating chlorite consumed to aldose oxidized for specific experimental conditions and (2) by the development of an analytical expression from kinetic and stoichiometric considerations. The former procedure is more accurate, but the analytical expression is more flexible, as it applies over a range of experimental conditions. However, either procedure gives experimental values for glucose within a few percent of the theoretical value, the accuracy depending on the amount of glucose present.

The kinetics of the oxidation of glucose have been studied over the experimental ranges of 3.4 to 4.4 pH, 30° to 65° C., 0.005 to 0.15 molar sodium chlorite, and 0.00006 to 0.0016 molar glucose. The reactions were followed by determining the change in chlorite concentration, as shown by iodometric titration and by photometric measurement of the soluble yellow gas, chlorine dioxide, formed in the oxidation of aldose by chlorite. Cellobiose, melibiose, maltose, and lactose were all oxidized at about the same rate as glucose, but nonreducing sugars and sugar acids were not appreciably oxidized under the conditions of the experiments.

NOTE: For further technical details, see "Determination of Glucose by Means of Sodium Chlorite," by Herbert F. Launer, William K. Wilson, and Joseph H. Flynn, *J. Research NBS*, Vol. 51, No. 5 (November, 1953).

THE FRANKLIN INSTITUTE

MINUTES OF THE ANNUAL MEETING
January 20, 1954

The Annual Meeting of The Franklin Institute was held on Wednesday, January 20, 1954, in the Lecture Hall. S. Wyman Rolph, President, called the meeting to order at 8:20 P.M. Approximately 365 persons attended.

The President asked for approval of the Minutes of the Stated Meeting in November, which were printed in the December JOURNAL. There being no corrections or additions, they were approved as submitted. The President suggested that those who had not read the Medal Day speech of William Francis Gibbs, Franklin Medalist, which also appeared in the December JOURNAL, do so for they would find it interesting—and humorous. He stated that approval of the Minutes of the Stated Meeting in December, which were printed in full in the January JOURNAL, would be deferred until the next meeting because the JOURNAL had just been issued and most members had not had a chance to read the Minutes.

He announced that the Annual Report of the Board of Managers will be presented at a meeting in the spring after receipt of the Auditor's report.

The President then called for the report of the Tellers appointed in accordance with the By-Laws to count the votes for the eight Managers to be elected. George S. Crampton, Chairman, reported that a total of 914 ballots were cast, of which 900 were legal and 14 defective, and that the nominees received votes as follows:

Edward G. Budd, Jr.	899	Richard T. Nalle	899
W. Laurence LePage	897	Charles S. Redding	898
Hiram S. Lukens	897	John Russell, Jr.	896
John S. Malick	898	William M. Scott, Jr.	897

The President declared that the above members were hereby elected Managers of the Institute for a term of three years. He thanked George S. Crampton, Joseph S. Hepburn, and Howard Stoertz for their services as Tellers.

The Secretary reported the following elections to membership for the month of December:

Active	29
Associate	35
Student	21

and a total membership, including Company and Library privileges, of 6,854 as of December 31, 1953.

The Secretary further reported that the Annual Report of the Trustees of the Elliot Cresson Medal Fund, as required by the Deed of Trust, would be deferred until the next meeting.

The President introduced Dr. A. E. Alexander, Gem Expert of Tiffany & Company, New York, who spoke on "Precious Stones and Pearls." He discussed the techniques used in the Tiffany laboratory for distinguishing synthetic rubies, sapphires, emeralds, etc., from genuine stones, as well as the methods employed for quickly and accurately differentiating cultured from natural pearls. The talk was most interesting and illustrated with beautiful colored slides of many unusual and famous gemstones.

The meeting adjourned at 9:35 p.m. with a rising vote of thanks to Dr. Alexander.

HENRY B. ALLEN
Secretary

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A. M. until 5 P. M. on Mondays, Tuesdays, Fridays and Saturdays; 2 P. M. until 10 P. M. on Wednesdays and Thursdays.

RECENT ADDITIONS

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JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

MOON, PARRY AND DOMINA EBERLE SPENCER: Interpretation of the Ampere Experiments.

POWLEY, HOMER S.: Rapid Estimation Method for Long-Pointed Projectile Trajectories.

HUFFMAN, DAVID A.: The Synthesis of Sequential Switching Circuits.

SWANN, W. F. G.: On H. Alfvén's Theory of the Effect of Magnetic Storms on Cosmic Ray Intensity.

MOON, PARRY AND DOMINA EBERLE SPENCER: The Coulomb Force and the Ampere Force.

STIPPES, M. AND R. E. BECKETT: Symmetrically Loaded Circular Plates.

ALBRECHT, H. O. AND C. E. MANDEVILLE: The Storage of Energy in Some Activated Alkali Halide Phosphors.

TAYLOR, WILLIAM C., JR.: A Neglected Method for Resolution of Polynomial Equations.

PAI, S. I.: On the Flow Behind Axially Symmetrical Attached Curved Shock.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, JANUARY 20, 1954

ACTIVE NON-RESIDENT LIFE

W. Wesley Hicks

ACTIVE FAMILY

Edwin J. Burgess, Jr.
Lincoln P. Dieffenbach
Edward J. Dwyer
Courtney V. Franklin
William K. Lauth
John Reichert
Abraham J. Rosenfeld

ACTIVE

Philip M. Alden
Beauveau B. Beals
John R. Carlson
Samuel Druckman
Albert W. Friend, S.D.
J. Aubrey Hope
E. H. L. Johannes
John W. Lang
A. Lewis Lichtenstein
Alex R. Lindsay
William J. McLaughlin, Sr.
Arthur E. Pringle II
Frederick A. Rojak
David W. Shearston
Frank C. Shermer
Mrs. Belle S. Starsmann
Eunice Stockwell, M.D.
Maurice C. Wyatt

NECROLOGY

David M. Bullowa '46
Robert W. Desing '43
Joseph B. Donnelly '52
Raymond S. Henning '51
Robert A. Millikan '37
Ph.D., Sc.D., LL.D., D.H.C.
Marshall S. Morgan '14
Charles E. Scheidig '52
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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

THE VARIATION OF MAGNESIUM REQUIREMENT FOR OPTIMAL ACTIVATION OF ALKALINE PHOSPHATASE

BY

M. H. ROSS AND J. O. ELY

When the concentration of the substrate was lowered, the pH for optimum alkaline phosphatase activity was found to be lowered (1). The question arose whether the substrate concentration influenced also the amount of magnesium necessary for optimal activation. The magnesium required at high and low initial substrate concentrations

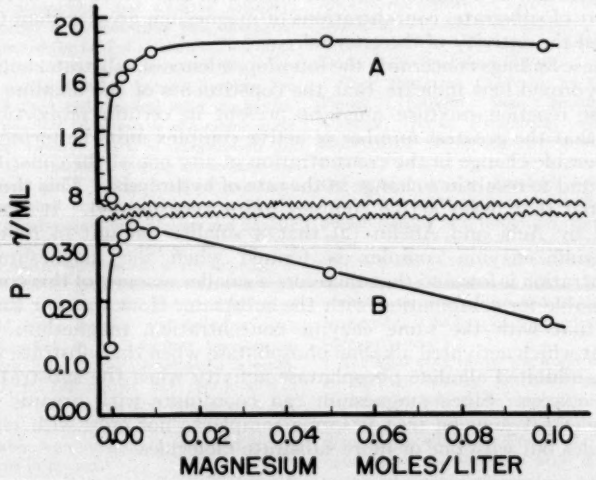

FIG. 1. Effect of magnesium concentration on alkaline phosphatase activity: curve A, Sodium β glycerophosphate, 10 mg./ml. at its pH optimum of 9.1; curve B, 5 γ/ml. at its optimum pH of 7.5.

was determined and only a slight change in the amount of magnesium required for optimal activation was found in comparison to the large change in substrate concentration.

Solutions of rat intestinal phosphatase were prepared according to the method described (1). Sodium beta glycerophosphate at different concentrations was hydrolyzed by the same preparation and concentration of the dialyzed solution of alkaline phosphatase, in the presence of $MgSO_4$ in varying amounts.

With a substrate concentration of 10 mg./ml. at the optimum pH of 9.1, the lowest magnesium ion concentration for optimum activation

was 0.015 M; with a substrate concentration of 5.5 mg./ml. at the corresponding optimum pH of 8.9, it was 0.0075; and with a substrate concentration of 5 gamma/ml. at the optimum pH of 7.5, it was 0.005 M (Fig. 1). Even though the substrate concentration was lowered by a factor of 2000, the magnesium requirement for optimal activation was lowered by a factor of 3.

The concentration of magnesium required for optimal activation of alkaline phosphatase, 10 mg./ml., extended over a wide range, 0.015 to 0.08 M. The concentration of magnesium required for optimal activation at substrate concentration of 5.5 mg./ml. extended from 0.0075 to approximately 0.02 M and then decreased. On the other hand the concentration of magnesium for optimal activity at the low substrate concentration of 5 gamma/ml. was limited to only 0.005 M. With this amount of substrate, concentrations of magnesium greater than 0.05 M reduced the activity of the enzyme.

These findings concerning the interdependence of substrate, activator and hydroxyl ions indicate that the constituents of the alkaline phosphatase reaction mixture must be present in certain proportions in order that the greatest number of active complex units be formed. A considerable change in the concentration of any one of the constituents was found to result in a change in the rate of hydrolysis. This therefore indicated that fewer active complex units were formed. It was suggested by Aebi and Abelin (2) that a smaller amount of reversible magnesium–enzyme complex is formed when the magnesium ion concentration is low and that therefore a smaller amount of this complex is available for combination with the substrate. However, our findings show that with the same enzyme concentration, magnesium in an amount which activated alkaline phosphatase when the substrate was in excess, inhibited alkaline phosphatase activity when the substrate was not in excess. Since magnesium can coordinate with organic phosphates (3), it may be that it forms complexes not only with enzyme molecules but with one or more substrate molecules.

SUMMARY

The amount of magnesium necessary for optimal activation of alkaline phosphatase at different concentrations of glycerophosphate substrate was found to vary. Decreasing the substrate concentration by a factor of 2000, decreased the magnesium requirements for optimal activation by a factor of only 3. At very low substrate concentrations, the amount of magnesium greater than the amount necessary for optimal activation, reduced the amount of enzymatic activity.

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BOOK REVIEWS

INTRODUCTION TO AERONAUTICAL DYNAMICS, by Manfred Rauscher. 664 pages, illustrations, 17×25 cm. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd; 1953. Price, $12.00.

Practicing aeronautical and mechanical engineers should find this volume extremely useful—particularly those working specifically in mechanics, aerodynamics, aeroelasticity, structural dynamics, automatic control equipment, and airplane stability. Noteworthy is the unusually well integrated treatment of the field of mechanics; too often custom in the past has prevented the recognized mutual meeting ground between the fields of fluid and classical solid mechanics.

Aeronautical Dynamics is the culmination of over twenty years of preparation by Professor Rauscher. During its writing the book was issued in the form of piecemeal mimeographed notes to students at M.I.T. taking a two-year course (Junior and Senior) in the subject. The arrangement of the subject matter corresponds to the sequence in the classroom presentations. Excellent illustrative examples are given which tend to hold one's interest and broaden one's understanding of the various phenomena. Although little of the presentation is essentially new, the author has coordinated a wealth of information and background material into this one volume; it should remain invaluable regardless of current and future advances in the art.

The text is divided into fourteen chapters. They include (in order of presentation) kinematics of a point, dynamics of a particle, dynamics of groups of particles, fundamental equations of fluid motion, stream function and velocity potential, fluid motion about simple bodies, transformations, airfoils, viscosity, kinematics of rigid bodies, dynamics of rigid bodies, and oscillations of systems with one or more degrees of freedom.

SAMUEL M. BERKOWITZ

KOSMISCHE STRAHLUNG, edited by Werner Heisenberg. Second edition, 620 pages, illustrations, 17×24 cm. Berlin, Springer-Verlag, 1953. Price, DM78.

Arnold Sommerfeld's seventy-fifth birthday in 1943 was to be marked by the publication of Werner Heisenberg's *Cosmic Radiation*. But on the very day that the first edition was to commemorate, bombs fell on Berlin. The attack destroyed the plates from which the book had been printed and burned to ashes almost all of the printed volumes. Only a few advance copies were saved. One of them, belonging to Professor S. Goudsmit, was used to prepare the well-known translation for Dover Book Publishers in 1946.

Since then, the Colloquia at the Max-Planck Institute were resumed. The Director, Werner Heisenberg, says in the foreword to the second edition that "each passing year since the end of the war has seen the publication of hundreds of papers, reporting on new experimental methods or on deeper insights into theoretical relationships. Thus it became necessary to re-write the first edition to do justice to the wealth of steadily accumulating material. The impression given by the first edition had to be modified frequently; it had to be changed entirely in some instances."

The final work brings the world up-to-date on matters relating to cosmic rays. The introduction reflects the more mature outlook of 1953. It is truly a "Survey of the Present State of Knowledge on Cosmic Rays." The most recent studies leave little doubt that cosmic rays carry a message from outer space, from the surface of the stars in our own galaxy.

The first chapter describes the "Origin of Cosmic Rays" whose primary component is the main clue to their nature. Electron counters and photographic emulsions have been perfected to a point where the interpretation of data requires an improved theory. Therefore, the first paper discusses "The Motion of Charged Particles in the Earth's Magnetic Field." The next paper gathers "Empirical Data for the Primary Component." The third paper discusses the observed "Periodicities" and the rare bursts

which are now known to be due to solar activity. The last paper deals with the "Various Theories on the Origin of Cosmic Rays."

The second chapter discusses "Nucleons and π-Mesons." The primary component arriving at the earth contains primarily nucleons, for example, protons or neutrons. As these particles first enter our atmosphere, they collide with oxygen or nitrogen nuclei and create many types of secondary rays. Within a certain energy spectrum, the nucleon component is responsible for most of the secondary effects. Physical measurements of the nucleon component are made with Wilson and ionization chambers, electron-counters, or photographic emulsions. Twelve papers compose this chapter: (1) The Nucleon Component in the Atmosphere, (2) Stars in Photographic Plates, (3) Penetrating Showers, (4) Collisions in Ionization Chambers, (5) Empirical Data on the Properties of π-Mesons, (6) π-Mesons and Gamma Rays, (7) Theory of Explosive Bursts, (8) Cascades within the Atomic Nucleus, (9) Disintegration of Nuclei in Photographic Plates, (10) Theory of Disintegration Processes During the Explosion of Nuclei, (11) Neutrons in the Atmosphere, (12) Heavy Unstable Particles.

The third chapter deals with "μ-Mesons" whose existence was first postulated by Nedermeyer and Anderson. The first paper in this chapter contains "A Historical Review." It is followed by a collection of material on the "Mass Determination of the μ-Meson." The final mass arrived at is 210 m_e. The third paper discusses "Decay and Absorption of μ-Mesons" by way of the energy spectrum, the mean decay time, and the absorption products. The fourth paper outlines "The Theory of μ-e Decay and μ-Absorption." The fifth paper treats "Collisions of Fast μ-Mesons with Nuclei." The sixth paper describes "μ-Mesons in the Atmosphere." The spectrum of the hard component of cosmic rays is best known at sea level where it consists mostly of μ-mesons. The fraction of nucleons in the hard component increases with altitude, however. According to present day knowledge, the μ-component vanishes at the edge of the atmosphere. Proof is obtained by filtering out the μ-component from the observed hard spectrum.

The fourth chapter is entitled "The Electron-Photon Component." Among the products of collisions between the primary particles and atomic nuclei are the μ^0-mesons. They decay rapidly into two gamma quantums. At higher altitude, they form the main source of the electron-photon component. Not so at low altitudes and below sea level. There, the μ-mesons are primarily responsible for the creation of the electron-photon component. Proof is found in the energy spectra of the π^0- and the μ-mesons obtained at various altitudes. These developments are traced by five papers: (1) Survey of the Electron-Photon Component, (2) Reactions Caused by Electrons and Photons, (3) Recent Results of the Cascade Theory, (4) Empirical Material on Electrons and Photons, (5) Forked Tracks Caused by Electrons.

"Interaction of the Components" is the title of the last chapter. Experimental methods as well as theories about the components are now unified. A nucleon cascade begins as primary particles enter the atmosphere and collide with nitrogen or oxygen nuclei. The first impact creates secondary particles, each of which produces new particles by further collisions. High-energy particles exhibit little angular scattering; the cascade grows, therefore, in the direction of the first primary particle. Thet otality of all participating particles consists mostly of nucleons for low-energy primaries. For higher energies, however, the relativistic time dilation of the half life of the various mesons lets them intervene to a larger degree in the cascade. Details of this interplay between the slow and fast components are outlined in eight papers: (1) A Qualitative Survey, (2) The Theory of Nucleon Cascades, (3) The Development of the Nucleon-Meson Component in the Atmosphere, (4) A General Theory of Electron-Photon Cascades, (5) The Development of the Electron-Photon Component in Air, (6) Experimental Data on Large Showers in Air, (7) A One-dimensional Theory of Large Showers in Air, (8) Angular and Spatial Distribution of the Electron Component of Large Showers in Air.

The authors have referred some choice mathematical morsels to a special appendix. The various problems met with here are: (1) A Theoretical Foundation for Dispersion

and Deceleration of Charged Particles, (2) Dispersion and Deceleration of High-Energy Electrons, (3) Experimentation and Evaluation on Measurements of Stars in Photographic Emulsions, (4) Additional Material to a Theory of Electron-Photon Cascades and the Large Air Showers, (5) the Classic Field Theory for Mesons, (6) A Quantized Field Theory for Mesons, (7) Formulas, Tables, and Graphs.

Cosmic radiation is a fascinating subject. It challenges the imagination through experiments performed with high-energy particles. The rays deliver a decipherable message from the activity on the surface of stars in intergalactic space; they permit the physicist to experiment with energy levels that outweigh the rest mass of the heaviest elementary particles. Werner Heisenberg's present volume is the expression of one of the great minds of this century. It makes an outstanding contribution to this important field of modern physics. It symbolizes the proving ground on which we gather our knowledge of the universe.

CARL HAMMER

ALGAL CULTURE: FROM LABORATORY TO PILOT PLANT, edited by John S. Burlew. 357 pages, diagrams, 17×25 cm. Washington, Carnegie Institution (Pub. No. 600), 1953. Price, $1.25.

As the attention of the Western Hemisphere has become focused more sharply on the East, we have become much more conscious of the problem of producing sufficient food for all of the Earth's rapidly growing population. Along with this problem, we have begun to think about our dwindling sources of energy, and literally turned our eyes to the sun. In seeking answers to both of these basic problems, quite a few scientists, encouraged by the Carnegie Institution of Washington, have considered and begun to investigate one of the simplest forms of plant life, the unicellular algae.

The cultivation of algae offers several advantages over our usual plant life, with respect to efficiency of growing and quality of food produced. Each microscopic cell is completely edible, and since the plant is merely a myriad of these unit cells, there is no waste. This fact should be compared with the fact that most of our domestic vegetables are less than 50 per cent edible (that is discarding roots, leaves, etc.). Furthermore the material is high in protein and essential amino acids, and in general highly nutritious. And more important, the algae can be grown almost anywhere in the world, regardless of terrain and climate, since they can be raised completely artificially in tanks and other chemical engineering equipment.

With regard to utilization of solar energy, algae are far more efficient than any of our conventional fruits and vegetables. They could conceivably be used as a source of chemical energy as well as food, by using heat obtained from burning the dried product.

This book on algae consists of a collection of original research papers dealing with the results of numerous experimental investigations. It presents the facts previously mentioned and others, in considerable detail. There is no particular attempt at continuity; the book must be considered as a scientific journal on a specific phase of research. The contributing institutions include Carnegie Institution, the Solar Energy Research Group in the Netherlands, The University of Tokyo, Arthur D. Little, Inc. and others.

Undoubtedly, an important criterion of the probability of success of algae as a food is its palatability. Tests which include "gag factories," are fairly encouraging. In the lighter vein, *Chemical and Engineering News* (**31**, 34, 3518, 1953) reports on a brief and admittedly meaningless investigation of the Washington Daily News with regard to this factor of palatability. The summary includes comments, of tasters, on algae à la Chinese pastry (good), algae cooked with rice (tasted like hay) and algae in a sauce for shrimp and chicken (not bad).

We believe this book brings the first reports on what may evolve into a partial answer to the problem of that large segment of humanity which encounters hunger or famine as a normal part of its existence.

ALBERT L. MYERSON

PRINCIPLES OF AUTOMATIC CONTROLS, by Floyd E. Nixon. 409 pages, diagrams, 15×22 cm. New York, Prentice-Hall, Inc., 1953. Price, $9.35.

We are learning more and more that the analytic processes developed several decades ago for studying electrical communications systems have even broader application than

was originally suspected. These principles have only recently engaged the interest of mechanical and other engineers, whereas they were formerly the fruitful province of the electrical specialist alone.

For example, it is interesting to speculate on the reasons which led to the discovery of the Nyquist Criterion, by a man studying telephone circuits, instead of by the mechanical engineers who have attempted to analyze myriads of hydraulic and other control systems. The electrical engineer has often led the way in developing the important techniques for analyzing complex control and instrument problems.

Floyd Nixon's *Principles of Automatic Controls* is a careful collection of principles and notation methods used to understand control problems. The author stresses the importance of system analysis by mathematical methods, devoting much of his book to demonstrations of stability relations in servomechanisms. All of the material covering these topics is treated in a completely understandable manner in spite of its highly mathematical character.

Besides the admirably adequate treatment of system principles and their mathematical analysis, Mr. Nixon undertakes an abridged description of some of the automatic computers which make the solution of control problems less formidable. This chapter races through the basic conceptions of both digital and analog computers giving the highlights of their use. Although certain machine arrangements capable of solving the example equations are shown, the discussion is really too scant for the reader to gain more than a respectful curiosity about computing facilities in general.

In evaluating the usefulness of this volume it can be said that a favorable comparison may be made with books such as Ahrendt and Taplin's *Automatic Feedback Control* or Chestnut and Mayer's *Servomechanisms and Regulating System Design*. The reader should remember that it is difficult to write what might be called a comprehensive text on the subject of automatic controls because so much territory must be covered. Most authors have had experience in only a few narrow aspects of practice, or they have studied the basic principles of servomechanisms. Judging from Floyd Nixon's book, his specialty has been the study of stability criteria in control systems, and he has held closely to the subject implied by the title of his writings, "principles."

There is little doubt that this book will do its part in aiding the process that was mentioned at the beginning of this criticism, the spread and clarification of the analytical methods used by electrical engineers for the benefit of men in other fields.

C. W. HARGENS

INSTRUMENT ENGINEERING, Volume II, by C. S. Draper, W. McKay and S. Lees. [Methods for Associating Mathematical Solutions with Common Forms.] 827 pages, diagrams, 23×28 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $15.00.

Instrument Engineering, one of the McGraw-Hill Publications in Aeronautical Science, appears in three volumes. The first, *Methods for Describing the Situations of Instrument Engineering* (or alternately, *Fundamentals*), develops a system of concepts, notation, diagrams and mathematical forms for describing the performance of physical devices. This second volume is about three times as large as the first and deals exclusively with mathematical methods of solution. The third volume is scheduled for release in 1954. Entitled "Applications of the Instrument Engineering Method," it is made up of examples that use the methods of Volumes I and II. The examples are based on meters, measuring system components, control system components, computing system components, regulators, servomechanisms, and feedback control systems.

The function of Volume II is to develop generalized solutions for integro-differential equation forms produced by the methods of Volume I. It is emphasized that mathematical reasoning will not produce a correct result from an equation that does not truly describe the physical situation under consideration. The material is presented in a sequence of chapters that deal first with the reduction of equations to standard nondimensional forms and then describe methods for finding solutions. Numerous specialized mathematical and graphical methods of solution are applied to moderately simple problems with sufficient completeness that more complex problems may be undertaken without difficulty. Although this volume is

a masterpiece in revealing the best methods of solving numerous forms of equations by the best particular method, it will not replace standard textbooks which specialize in standard methods of solution.

Much of the mathematics will be unfamiliar to the engineer who has not gone beyond the first course in differential equations. Some difficulty may arise if the reader is not familiar with the notation employed. These are not criticisms of the book but warnings to the reader. Engineers and students who are willing to become familiar with the material will be able to select an effective method for solving any given equation. In addition, their activities as engineers will be facilitated by the ability to discuss problems with colleagues whose training restricts them to the viewpoint of a single mathematical method.

Instrument Engineering is being reproduced by offset printing which gives great flexibility in composition. The mathematical notation, the tables, and the numerous detailed summaries are extremely well planned and easy to follow. The illustrations are profuse and well chosen. *Instrument Engineering* has been built up during a period of over twenty years of intensive experience with teaching and practice at the Massachusetts Institute of Technology. It should be an effective advanced textbook and basic reference for students and engineers who are interested in the mathematical background of measurement and control. A. DONALD HAY

NUCLEAR PHYSICS, by W. Heisenberg. 225 pages, illustrations, 13×20 cm. New York, Philosophical Library, 1953. Price, $4.75.

Writing is an art that comparatively few people acquire. For some curious reason the scientists of the old world are more proficient in this art than Americans and in no book is this theme better illustrated. Writing with a profound lucidity which is a model of exposition, Dr. Werner Heisenberg, one of the outstanding physicists of our time, brings to life a story of man's struggle to tame the atom. There are books without number on nuclear energy and this reviewer has met up with a great many, but nowhere is the subject treated in simpler language without the handicap of wading through the mathematical symbolism indigenous to the subject.

Nuclear Physics opens with a short history of atomic theory from antiquity to the close of the nineteenth century. There follows a short essay on molecules and atoms, preparing the reader for the principal topic of the book which includes radioactivity, the binding energy of the nuclei, nuclear structure, artificially induced transmutations. The book concludes with an appendix on German researches in atomics during the last war.

This book like so many other recent books is based on a series of lectures and is pointed to those who though lacking previous training in theoretical physics are still familiar with the basic physical principles. Unlike the other books, there is a continuity which ties the various chapters together in a natural sequence. It is highly recommended.

I. M. LEVITT

MICROWAVE SPECTROSCOPY, by Walter Gordy, William V. Smith and Ralph Trambarulo. 446 pages, diagrams, 16×24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $8.00.

The work spectroscope usually brings to mind an optical instrument containing a prism or grating.

A microwave spectroscope is very different. It is composed primarily of a radio transmitter and receiver. The region of the spectrum usually considered as microwaves extends from 1000 megacycles to 300,000 megacycles. Contrast this with the 1 megacycle frequency of the middle of the broadcast frequency and it is obvious that the radio equipment involved has little similarity to usual electronic equipment.

The first microwave spectrum was obtained in 1934 when Cleeton Williams obtained the microwave spectrum of ammonia. The radar developments of World War II gave strong impetus to the field. It was a surprise to the reviewer how much the technique has grown in the intervening years. Data on the molecular structure of over 84 molecules have been obtained by microwave analysis. Through a study of the hyperfine structure observed at low temperatures, many nuclear properties have been determined.

The book is divided into three main parts: the first part is devoted to instrumentation and experimental techniques; the second, to theory of molecular structure and the resulting microwave spectra; and the last (but by no

means least), is an extensive compilation of data collected to date with microwave methods and a complete bibliography of the subject.

This book is not one which a spectroscopist or physical chemist could absorb with ease without developing a background in ultra-high frequency techniques. However, the possibilities of microwave spectroscopy in the fields of physical chemistry and chemical analysis make this a book which would be well worth their study.

L. C. CONANT, JR.

HIGHER TRANSCENDENTAL FUNCTIONS, Volume II, edited by A. Erdélyi. 396 pages, 16×24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $7.50.

This is the second volume of a trilogy inspired by the papers of the late Harry Bateman. The project is sponsored by the Office of Naval Research and the California Institute of Technology. Together, the three volumes aim to be the most up-to-date reference on higher transcendental functions.

The present volume has seven chapters, which are numbered VII through XIII, to preserve continuity with the first volume. Chapter VII on Bessel functions has two parts: theory and formulas. The influence of G. N. Watson and E. T. Whittaker is felt everywhere, but we find also very recent material such as MacRobert's remainder theorem for asymptotic expansions of Bessel functions of the third kind, or Schöbe's and Tricomi's asymptotic expansions for Bessel functions of the first kind.

The eighth chapter lists functions of the parabolic cylinder and of the paraboloid of revolution. We find integral representations, asymptotic expansions, and relations with the confluent hypergeometric functions. The highlight of this chapter is undoubtedly Cherry's theorem on functions of bounded variation in a finite interval that are absolutely integrable along the entire real axis.

Incomplete gamma functions are treated in another chapter. A section on error functions contains Tricomi's recent (1951) expansion of $erf(x)$, $erf(\sqrt{x})$, and $erfi(\sqrt{x})$ into series of Bessel functions.

The tenth chapter introduces orthogonal polynomials through the more general orthogonal functions. The classical approximation problem in L^2 and the mechanical quadrature problems are followed by the orthogonal or Tchebichef polynomials of Legendre, Gegenbauer, Jacobi, Hermite, and Laguerre. Among the properties dealt with are asymptotic behavior, zeros, and series expansions. The next chapter continues with spherical and hyperspherical harmonic polynomials. Many of the elegant proofs are taken from unpublished lectures of G. Herglotz. The twelfth chapter deals with orthogonal polynomials in several variables, in the triangle, the circle, and on the sphere.

The final chapter, too, has two parts. The first part on elliptic integrals begins with a brief historical note, followed by the theory of complex elliptic integrals and functions which is easily read by itself. We find details on periods and singularities, Riemann surfaces and branch points. An extensive table lists the transformations to Legendre's normal form for both real and complex zeros. The second part of this chapter deals with elliptic functions. The classical inversion problem leads to doubly periodic functions in general, Weierstrass functions, Jacobian elliptic functions, and special values. Included are the computationally important formulas of Tricomi for elliptic integrals of the third kind.

Like the first, this volume was set capably by the varitypist Rosemarie Stampfel. It contains over two thousand numbered formulas and six hundred references, mostly to European literature. American mathematicians and physicists will thus obtain a good view of Continental developments. Curiously enough, Harry Bateman's own "Guide to Tables of Bessel Functions" (Mathematical Tables and other Aids to Computation, July 1944) is not even mentioned among the references. Nor do we find Alan Fletcher s "Guide to Tables of Elliptic Functions" (Mathematical Tables and other Aids to Computation, October 1948) among the references to Chapter XIII. These serious omissions are in part compensated by the didactic value of the imposing number of explicitly stated relationships. Together with the third volume, due in 1954, *Higher Transcendental Functions* will be a tribute to the memory of the great Harry Bateman.

CARL HAMMER

AN INTRODUCTION TO POWER SYSTEM ANALYSIS, by Frederick S. Rothe. 187 pages,

diagrams, 15×24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $5.00.

This slim volume (187 pages) will be welcomed by engineers concerned with the planning and operation of electrical power systems. It is the first text to bring together, in complete yet concise form, the powerful methods of power system analysis developed in the last twenty years by S. B. Crary, Edith Clarke, C. Concordia and others.

The subject matter includes: the derivation of equivalent circuits, the per unit system, synchronous machine theory, symmetrical components, the $\alpha\beta 0$ components (to which Mr. Rothe has given the name of Clarke components, in honor of Edith Clarke), and a thorough treatment of transient and steady-state power system stability. Mention is made of the indispensability of the a-c. and d-c. network analyzers in the analysis of large and complicated systems today. This is of immediate concern to public utility planning engineers, as witness the several recent installations of large a-c. network analyzers, the latest one at The Franklin Institute. BRUCE B. YOUNG

SPACE TRAVEL, by Kenneth W. Gatland and Anthony M. Kunesch. 205 pages, plates, 14×22 cm. New York, Philosophical Library, 1953. Price, $4.75.

In their preface the authors state: "With the progress that is now being made, it is possible to illustrate how space-flight may be expected to evolve in a series of logical and well-defined steps." Their book, *Space Travel* is dedicated to showing the "logical and well-defined steps" and they have succeeded in an authoritative and lucid fashion.

The authors, together with their colleague A. E. Dixon, have collaborated for the past six years in technical studies dealing with space travel. Their contributions have appeared in the *Journal of the British Interplanetary Society* and have been subjected to critical analysis. The results of their early efforts are reflected in at least one section of the book.

In seven chapters beginning with a carefully documented historical account, the book takes up in order the development of rockets culminating in the building of the V-2. The problems of space flight including the behavior of man in space are related in a stimulating fashion. Then the establishment of the space station as the base of operations for future travel to both the planets and the stars is delineated and the book concludes with this theme.

The book is profusely illustrated with a section in the back to serve as a further description. Mention must be made of the illustrations facing page 176 illustrating the minimum inter-orbital space-ship and the ion-drive space-ship. This is the first time that this reviewer, who has made a comprehensive survey of the field, has seen this vehicle illustrated. A bibliography completes the book.

In a sense this book complements Mr. Gatland's "Development of the Guided Missile," published in 1952, in which the rocket, as a weapon of the present and future, was described. Now, with the aid of Mr. Kunesch, the application of the rocket to the conquest of the space frontier is admirably covered. I. M. LEVITT

TEMPERATURE MEASUREMENT IN ENGINEERING, VOLUME I, by H. Dean Baker, E. A. Ryder and N. H. Baker. 179 pages, diagrams, 16×24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $3.75.

This is the first of two volumes on the general subject of temperature measurement to be presented by the authors. The first volume is concerned mainly with the thermocouple technique, while the second will cover such diverse subjects as the measurement of extremely high or low temperature, measurement of surface temperature, radiation pyrometry, etc.

The first chapters introduce the concept of heat and the usual methods used for its measurement. The authors discuss briefly the glass thermometer, the vapor pressure thermometer, radiation pyrometer, resistance thermometer, etc. They also describe their special uses and the possible errors that must be considered in their use.

The remainder of the book describes in detail various aspects of the thermocouple technique. Circuits are shown and methods for testing their accuracy are given. Full details are given for the installation of a thermocouple junction, even to the extent of describing drilling techniques. The last chapters deal with the several ways of cementing in the junction, and methods to

protect the junction from heat, moisture, etc. Finally, the authors discuss methods of determining a temperature gradient with a thermocouple. In all cases actual problems of measurement are discussed and solved.

In the preface the authors state that they wished to write a handbook on temperature measurement. In this, they have succeeded. The book is filled with a wealth of methods and techniques and extraneous descriptive material is kept to a minimum. This means, however, that the book is not especially easy to read, since one's attention is constantly being drawn to a new subject or method.

The book is extremely well referenced, with many of the references being quite new. Together with the future second volume, it should be useful to engineers and scientists who find it necessary to make precise temperature measurements.

ROBERT B. CAMPBELL

PROCEDURES IN EXPERIMENTAL METALLURGY, by A. U. Seybolt and J. E. Burke. 340 pages, diagrams, 16×24 cm. New York, John Wiley & Sons, Inc., 1953. Price, $7.00.

Many of us have found it necessary at some time to prepare an experimental alloy, fabricate it to sheet, rod or wire, and heat treat it for use in some specially designed experiment. Somewhere during this process we had to have help in simple laboratory procedure. It may have been in furnace design, choice of melting crucible, method of melting, etc.; but certainly there was some phase of the process that was unfamiliar to us. This book is designed for the express purpose of helping us in just such situations. It is a practical manual outlining metallurgical procedures used in preparing metals for laboratory experiments.

The arrangement is logical in that it describes the equipment and methods in the order in which they would be encountered if one were to prepare an alloy.

Resistance wire furnace construction is discussed in detail. The authors give the excellent advice that one of the simplest methods to build such a furnace is to utilize commercial replacement windings. This is often overlooked by the research worker and he usually wastes time and money by winding his own elements. Other standard ways of obtaining high temperatures are also discussed.

Measurement and control of temperatures are troublesome problems. It is a frustrating experience to lose temperature control and melt a finished group of alloys that were undergoing some less drastic heat treatment. Such experiences can be minimized by careful choice and design of measurement and control apparatus. The authors help to clarify this problem by devoting two chapters to it. A valuable table is given in Chapter Four listing refractories, their properties, uses, and the manufacturer. The value of such a table could be increased greatly by including information as to what the best crucible was for melting a particular metal. For example if one wanted to choose a crucible for melting iron, an examination of the table would give him the following list: alumina, magnesia, zircon, and zirconia. One of the best refractories for melting iron from the viewpoint of non-contamination is beryllia, yet this is not shown as being satisfactory for iron. Of the four tabulated, zircon is the poorest because of the instability of the silicate in the presence of iron even with small amounts of carbon. High purity alumina is the best; zirconia and magnesia follow. In fairness to the authors it should be pointed out that they state that a considerable difference of opinion exists on this subject.

A short section on the laboratory preparation of crucibles is included. This is necessary information for special refractories; however it should be emphasized that this is a process that is usually done more economically, faster, and better by some refractory company.

The chapter entitled "Vacuum Systems" should be of considerable value to the research worker. Although more comprehensive and elegant texts are available on this subject, the authors set forth the principles and practices in a concise and satisfactory manner. This chapter contains particularly good illustrations. The authors state that oil pumps generally require no cold traps. Although this is true as far as diffusion pump oil condensing in the melting chamber is concerned, good high vacuum practice requires a cold trap to freeze out condensables from the melt.

The methods of melting and heat treating are discussed thoroughly. This is material

which is likely to be of the greatest interest to the investigator since it describes in detail how to make an alloy. The authors describe melting and heat treating furnaces, procedures, gases in metals, protective atmospheres, casting techniques, quenching techniques, etc.

For those not familiar with fabrication problems, it often comes as a surprise to find that forming a cast ingot into a sheet, rod, or wire is a difficult procedure. The authors discuss rolling, swaging, wire drawing, and extrusion in enough detail to enable one to choose the proper method to satisfy his needs.

The remaining quarter of the book is devoted to powder metallurgy, the preparation of pure metals and single crystals. Again the authors give excellent advice in suggesting that, when possible, pure metals should be purchased rather than prepared in the laboratory. They also point out an important fact often overlooked by the user of "pure metals" and that is that a metal can be quite impure even if spectrographic analysis shows high purity with respect to metallic impurities. This of course is because of the non-metallics present.

A list of forty-two suppliers of commercially available pure metals is given, together with typical analyses of the principal impurities. Such a list is invaluable to anyone using pure metals. This list is not complete in that the supplier of the purest metal is not always given. This is understandable, since a correct list would require that the authors obtain samples of metals from all of the suppliers and analyze them. The analysis given by the supplier cannot be depended upon.

Methods of purification are outlined but unfortunately only half a page is devoted to the method of "zone melting." The importance of zone refining should not be underemphasized. The authors could well have devoted more space to it and included a drawing of zone refining equipment.

The final chapter on the preparation of metal single crystals, although brief, is adequate. This phase of metallurgy is becoming of increasing importance in research. There seems to be a considerable amount of skill and art necessary for the production of metallic single crystals which of course can be obtained by experimentation. Very seldom do detailed instructions and equipment specifications benefit the beginner appreciably. Once he is familiar with the general principals such as presented by the authors, he has to build and experiment with his own equipment in order to obtain the correct conditions for single crystal production.

This book does not attempt to present new, unusual, or specialized techniques. Rather, it outlines generally used laboratory methods for metallurgical procedures. It is written in simple straightforward terms with enough illustrations to lend clarity to the text. As such, it is a worthwhile addition to the library of any research group involved preparation of experimental metals and alloys.

R. L. SMITH

INTRODUCTION TO GEOMETRICAL AND PHYSICAL OPTICS, by Joseph Morgan. 450 pages, diagrams, 16×24 cm. New York, McGraw-Hill Book Co., 1953. Price, $6.50.

The presentation of physical optics and geometrical optics in one volume is the chief accomplishment of the author of this book. He intends the book to be used in a one semester optics course for sophomores or juniors. Of necessity he had to simplify both fields and eliminate some subjects. This reviewer has been using this book for reference on an optical project for six months. So far, he has found it lacking only in such topics as Photoelectric Emission and Metallic Reflection in Metals, both topics of interest only for fundamental research and design. More general topics are well covered in the text. The explanations are clear and concise. For mathematical proofs, the author presents the non-calculus proof on the left side of double column pages and the calculus development on the right hand column. This treatment is very refreshing. Many familiar proofs gain new clarity when the calculus proof and the algebraic proof are studied simultaneously.

An appraisal of this book from a pedagogical viewpoint is beyond the experience of the reviewer. However, for personnel working in spectroscopy, and other fields of applied optics, the book should be a welcome guide to the newcomer, and a good, one volume, reference book for the experienced worker.

L. C. CONANT, JR.

SCIENTIFIC AMERICAN READER, compiled by the editors of *Scientific American*. 626 pages, diagrams, 16 × 24 cm. New York, Simon and Schuster, Inc., 1953. Price, $6.00.

Five years ago the century "old" *Scientific American* was bought by an astute young man whom this reviewer knew as the Science editor of *Life* magazine. Gerard Piel gathered around him some of the finest science reporters and writers in the country and the "new" *Scientific American* was born. By skillful means and sustained effort the format and content of the magazine were changed until it became the dominating agency for science dissemination in this country with a tremendous potential for influencing and moulding public opinion.

Its prime position is not one which just happened—an extraordinary volume of work went into the shaping of its structure. As an example, the *Scientific American* is the only magazine to submit to its readers an extensive query to uncover the fields, professional treatment, illustrations and content its readers desire. With consummate skill the magazine is being jockeyed into the enviable position of the authoritative spokesman of science.

The above is an introduction to the *Scientific American Reader*, a collection of 57 contemporary articles divided into 12 categories of major interest. The authors are scientists working in the particular field about which they write and have attempted to convey their contribution with a clarity, precision and completeness in the finest sense of exposition.

The topics covered present science arrested in a moment of action and range from the macrocosmos to the microcosmos—from the origin of man to the thinking machines—from atomic energy to medicine. It would dilute the purpose to attempt to cover too much of science so a rational selection was made to incorporate those items of popular interest which will promote the wider understanding and the acceptance of science by an alert society.

The list of authors is an index of the caliber of the book. In astronomy Whipple, Bok and Lovell are top men in their fields. Marshak, Bethe, Kamen and Ridenour are physicists of the first magnitude. Right on through the list are names like Rabinowitch, Dobzhansky, the Morrisons, Krogman, Stern, Ittleson, Kilpatrick—every man a leader in his field and every man an articulate member of his own discipline who can write with a profound lucidity.

The *Scientific American Reader* is the finest book of its type and for many years will be welcomed as a stimulating reference book for many fields of science. This reviewer has jealously guarded his *Scientific American* files; now he possess the cream of the files in book form.

I. M. LEVITT

The Editors wish to emphasize that the JOURNAL is copyrighted and no part of any issue may be reproduced without our permission. Recently a trade publication reprinted material from our *Laboratory Notes* for July 1953, without such permission. Since, in this particular instance, no credit was given to the JOURNAL, to The Franklin Institute Laboratories, or to the author, the readers of the trade publication were erroneously led to believe that the material was being published for the first time.

CURRENT TOPICS

First "Surface-Barrier" Transistor. —Development of a new type transistor which out-performs all transistors currently in use for both military and civilian equipment, was announced recently by Philco Corporation.

"The new 'Surface-Barrier' transistor," said Leslie J. Woods, vice president-director, Philco research and engineering, "operates at high frequencies and with low power consumption—requirements which have limited the use of transistors up to this time to hearing aids and devices where stability is relatively unimportant.

"The 'Surface-Barrier' transistor is the most important advance in electronics since discovery of the point-contact transistor. It opens up an entirely new realm for transistors in both military and civilian applications."

The "Surface-Barrier" transistor was announced to The Franklin Institute at a meeting attended by members of the Institute, the Institute of Radio Engineers, representatives of the Department of Defense and officials from other companies engaged in electronics research and production.

"The 'Surface-Barrier' transistor is of unique construction and unprecedented performance," Mr. Woods said. "It has operating characteristics which make possible its use at frequencies 10 to 100 times as high as obtainable with the older alloy junction transistors.

"The new unit operates with such economy of power consumption that for the first time a portable military communications receiver operating on very high frequency channels can be powered solely by two flashlight cells. The receiver can be made as small as a pack of cigarettes and has an operating life of many weeks," he said.

Behind the "Surface-Barrier" transistor is a new method of processing germanium which gives promise of transistor mass production, Mr. Woods said, because the method may be precisely controlled. Inability to control production methods has been a limiting factor in the production of transistors up to this time, he added.

"This new method exhibits the highest mechanical precision yet attained in machining germanium," Mr. Woods said. "The process consists of directing two shiny streams of liquid indium salt at opposite sides of a tiny slab of germanium. Electric current is passed through the streams so as to etch away the germanium. This process continues until the two streams almost drill through the slab. When the germanium has been etched down to a few ten thousandths of an inch in thickness, the current is suddenly reversed. The etching is thus instantly arrested and indium is immediately electroplated on the germanium by the reversed current flow to form electrodes on both sides. The resultant assembly, with wires attached to the two electrodes, is hermetically sealed in a metal container.

"Methods have been developed which arrest the etching so precisely that the remaining thickness of germanium is controlled to a tolerance of 10 millionths of an inch, or less than the wave length of visible light. Such precision has not been possible in producing earlier types of transistors."

Mr. Woods said "Surface-Barrier" transistors operate reliably in frequency ranges up to 70 megacycles, which include an important military VHF communication band from 20 to 58 megacycles.

Automatic Inspection Device.—General Electric Company engineers have developed a new automatic inspection device to speed production and insure uniform quality of sheet electrical insulating materials.

It was designed and built by the company's General Engineering Laboratory to find, "measure" and mark voids in such materials faster and more accurately than the best-trained human inspector.

In a year's test here, it enabled a 260 per cent boost in production of mica tape, which is in great demand as insulation in motors and generators.

Accompanying benefits included an average saving of $100 per eight-hour shift in material costs as the result of the constant check on quality.

Mica tape is made from splittings about the size of a half dollar. These are blown into a chamber and fall like snowflakes onto a steel drum. The drum in turn deposits the flakes on a moving web of varnish-soaked paper.

Since the flakes do not fall uniformly, open spaces are hand-patched as the tape moves past a group of operators.

Then the tape passes through the inspection device, which gives a continuous indication of the quality as determined by the number and size of voids still remaining. If quality falls below an established level, the device sounds a signal and corrections are made.

A permanent record of quality is obtained by means of a photoelectric recorder. In addition, the device is designed to mark with a drop of dye all voids that exceed a permissible size.

Detection elements "scan" the entire sheet as it moves past the point of inspection. When a void passes between the detection elements, current flows. Inspection is accomplished by automatic measurement of the flow of current, which is limited to low value and cannot harm human beings or ignite volatile vapors.

Study to Aid Courts in Cases Involving Scientific Questions.—How successful have this nation's courts and administrative agencies been in rendering correct decisions on complex scientific questions beyond the comprehension of well-educated jurists and administrators? Can new procedures be devised which will produce more reliable results?

A study of these questions will be made pursuant to the terms of a $50,000 grant made by Major Edwin H. Armstrong, well-known electrical engineer who made extensive contributions to radio, including the invention of FM radio.

Under the terms of the grant, the study must be formulated and directed by the dean of the Columbia University School of Law, in association with two or more co-directors selected by him. It is required that one director be an outstanding engineer and another an outstanding lawyer, both to be impartial persons not connected with Columbia University. Dean William C. Warren, Dr. Karl T. Compton, Bethuel M. Webster, and Professor Young B. Smith will serve as directors of the project, with John G. Palfrey in charge of research.

In making the grant, Major Edwin H. Armstrong explained that it was "for the purpose of making a study or studies of problems affecting the public interest which are of mutual

concern to the engineering and legal professions.

"It has been my observation over a long period of years," Major Armstrong said, "that public bodies, in order to discharge their functions, are frequently required to make findings of fact on technical and scientific matters that are beyond the comprehension of laymen, and that the techniques involved in the ascertainment of such facts have not been adequately developed, with the result that important decisions sometimes have been made, and important actions, taken, upon erroneous findings of fact in technical and scientific fields.

"The studies I have in mind would be directed to that problem. They would canvass the experience of particular public agencies of the United States with a view to determining how far they have been successful in the ascertainment of scientific and technical facts and to what extent, if any, the public interest has been adversely affected by incorrect determinations of such facts. Such studies might result in suggestions or recommendations for better techniques and procedures to be employed in the making of determinations of fact in scientific and technical fields," Major Armstrong concluded.

New Thermal Printer.—Smudge-free printing, high speeds and high reliability characterize the new Thermal Printer developed at the Research Center of Burroughs Corporation, Philadelphia, and shown for the first time at the Eastern Joint Computer Conference in Washington.

The Burroughs Thermal Printing Technique uses heated printing plungers to transfer a carbon coating to a paper copy. Using heated surfaces to make the transfer reduces the force required and effects clean copy at high speed.

Developed specifically for use with the Burroughs Laboratory Computer (a large-scale electronic "giant brain"), the Thermal Printer accepts a four-bit excess-three binary code and prints a choice of 16 characters in the 5×7 matrix form. (Other types of coder permit a multitude of character choices.) This is a straight serial printer that prints on $\frac{11}{16}$-in. wide tape at a maximum rate of 30 characters per second, 10 characters to a lineal inch. The size of the printed character is $\frac{5}{64}$-in. wide by $\frac{5}{32}$-in. high. The tape drive, selection and positioning apparatus cause the tape to stop and start each cycle; the printing takes place while the tape is at rest.

Perhaps the most important feature of the Printer is the simple and extremely accessible assembly that results from the use of the directly heated printing pins. The present Burroughs Thermal Printer is a laboratory developmental model which has been tested to 3,500,000 cycles in simulated computer operation. Plans are under way to operate the Thermal Printer as an output device on the Burroughs Laboratory Computer, which is currently operating as a Computer Service solving problems for industry.

Magnetic Tape Recording of Television Pictures.—Recording of television pictures on magnetic tape in color and in black-and-white was publicly demonstrated for the first time recently by the Radio Corporation of America at its Princeton laboratories in a preview of new techniques that will simplify the entire art of making pictures in motion.

Brigadier General David Sarnoff, Chairman of the Board of RCA, said

that this new method of recording sight is similar in basic respects to tape recording of sound. He described the achievement as the first major step into an era of "electronic photography," in which motion pictures in color or black-and-white will be produced quickly and economically, without any photographic development or processing.

General Sarnoff, declaring the development had far wider horizons than its immediate purpose in TV broadcasting, pointed out advantages of magnetic tape over ordinary film.

"Magnetic tape requires no chemical processing," he said. "The pictures can be viewed the instant they are taken, which adds new flexibility in making motion pictures. An unlimited number of copies of magnetic tape recordings can be made quickly. Recorded tapes can be preserved indefinitely for historic reference, or, if desired, can be electronically 'wiped off' and reused again and again."

RCA's method of video recording is similar, in basic respects, to the techniques used to record speech and music with present-day magnetic tape sound equipment. Electrical signals are impressed through a recording head—a small horseshoe electromagnet—onto the magnetically treated surface of a plastic tape. As the tape is drawn across the recording head, the head continuously changes the magnetic polarity of the magnetic oxide particles on the tape so that they become a compact code of the original signal.

For playback, the tape is drawn across the same, or a similar head. The magnetic "shorthand" on the tape causes an alternating current to flow in the windings around the reproducing head. The reproduced current closely duplicates the original signal.

Although the principles are similar, the engineering problems are not; audio recording is today an easy task compared with video recording. The reason is that audio signals are in the range of 20 to 20,000 cycles per second; while video signals range up to 4,000,000 cycles per second. And color television signals, as now formulated, must carry at least twice as much pictorial information as black-and-white. Besides, video tape must carry the associated sound signals.

In summarizing the present status of research and engineering problems as they relate to video tape recording, Dr. Engstrom, Vice President in charge of RCA Laboratories Division, said: "While some technical problems still must be surmounted before video tape equipment can be made available commercially, RCA considers that the toughest of them have been conquered and that further development is certain to solve the remainder."

Ultra-Sonic Burglar Alarm.—A new burglar alarm that capitalizes on the use of inaudible sound, traps intruders by their motion anywhere in a room or similar enclosed area. The system was invented by Mr. Samuel Bagno, vice president of the Alertronic Corporation, Long Island City, N. Y., and is manufactured by the company.

Operating on the principle of magnetostrictive vibrations, the supersensitive burglar alarm reports any disturbance of the ultra-sonic waves transmitted within an enclosed area.

The Alertronic system appears to be a simple device. Mounted on a wall, it looks like half a coffee can painted grey. From within the "can," which is really a diaphragm, ultra-sonic waves are generated. A similar "can" on an opposite wall

receives the waves. Any motion that disturbs the waves causes the receiving "can" to transform the sound into electricity, feed it into an amplifier and then sound the alarm.

The sensitivity of the system is controlled just as you would the volume of your radio. It is usually set to detect a body as small as a child's, but can be tuned to such high sensitivity that a deep breath, a wink of an eye, or wiggling finger will start it.

This sensitivity extends to the masses of air set in motion by fire. According to Mr. Bagno, the alarm can be set off simply by igniting a wad of newspaper. In Chicago, the ultra-sonic system detected a fire in an electroplating plant that was put out before any serious damage was caused.

Ultra-sonic vibrations are generated from four nickel rods, thin as wooden matches and twice as long. The rods are riveted to the bottom of the "can" and wound with copper wire to set up a magnetic field. As the rods shrink and relax, 19,200 times per second, they vibrate the "can" or diagram which sends out waves into the surrounding area. 19,200 cycles is beyond the hearing range but not so high that the air would absorb the waves.

The principle of the new system, magnetostriction, has been known to science for many years, but until recently it was regarded as a laboratory curiosity. With the development of magnetostrictive oscillators, new fields of study were opened to science. A number of metals have this magnetostrictive property that utilizes an alternating magnetic field to get contraction and elongation. Few of them have it to any commercially valuable extent. Most widely used is nickel because it has a combination of several essential properties. This is the principle on which Mr. Bagno began his work in 1941. He discovered that a wave could be made to bounce off a wall and cast an inaudible echo from any moving object.

The system not only detects burglars and fire but it has been found to kill mice and temporarily drive rats away. In a test by a Houston, Texas, school, six white mice caged near one of these devices were found dead in 30 minutes. In plants, mice were found with their skulls crushed, indicating they raced around in a frenzy until they hit something and died.

The Alertronic alarm system has received the approval of The Underwriters' Laboratories, who set standards for detection. Insurance underwriters allow up to 60 per cent discount on premiums where this system is used under strict conditions and in conjunction with a central station alarm system.

Totally-Enclosed Unit-Cooled D-C. Motor.—A new totally-enclosed, unit-cooled, d-c. motor for use in severe atmospheres has been announced by the General Electric Company's Direct Current Motor and Generator Department.

The new motor, smallest standard model of its type, was developed for application in machine-tool, paper, cement, chemical, rubber, and steel-mill industries, and materials-handling operations. Available in ratings from 15 to 200 hp., it features 50 per cent greater heat transfer in a 37 per cent smaller unit than previous designs.

Compact cooling units, mounted completely within the over-all length and width of the motors, permit operation at extremely low speeds for long periods of time because ventila-

tion is independent of motor speed, G-E engineers said.

A double system of blowers assures rapid cooling. One blower assembly forces cooling air from the room through the external passages of the cooler, while the other forces internal motor air through the closed internal passages of the air cooler.

The blower motors, standard G-E totally enclosed type, are for 3-phase, 60-cycle power supply. The blower controller consists of a simple magnetic switch which can, at a slight additional cost, be interlocked with the main controller so that both are started simultaneously.

A thermostatic relay protects the main motor in case of blower-motor failure. Relay contacts can be arranged to cut off the main motor or sound alarm. Located at the commutator-end air duct, the operating mechanism is accessible from the outside.

The new motors are available in the same speed and horsepower ratings as standard, constant or adjustable-speed, general purpose motors.

Chloromycetin Effectively Combats Scrub Typhus.—Scrub typhus, the disease that killed hundreds of Allied and Japanese troops in the Burma campaign during World War II, hasn't claimed a single soldier on active service against Malaya's Communist terrorists.

Captain D. H. de T. Reade, military information officer on non-tactical subjects attached to Headquarters Malaya in Kuala Lumpur, said the reason is "the wonder antibiotic Chloromycetin, which has proved 100 per cent effective in curing it."

He wrote a report published by Malaya's two leading newspapers, the *Sunday Gazette* and the *Sunday Times*, which said, "So swift is the cure that soldiers generally can be back on duty within a week. By older methods of treatment, they were away two to six months, and very often one in four died."

Scrub typhus is carried by microscopic mites, so small they look like specks of red dust, Capt. Reade explained. Troops operating against the Communist terrorists cannot avoid coming into contact with the mites, because much of their patrolling has to be done in cleared forest areas where the mites are so often found.

Captain Reade pointed out, "It was in Malaya that Chloromycetin was first used in the treatment of scrub typhus. In mid-1948, a small wooden box seven inches cube in size was brought to Malaya. It contained the world's supply of the drug and was to be used by the first of five U.S.A. Army Research Service teams to work in Malaya. The first soldiers treated were soldiers in Kuala Lumpur—five or six of them. It was one of the big dramatic moments in medicine."

He said the 1948 experiments were successful and added:

"Chloromycetin completely kills the fever within 36 hours. What is more, it also cures typhoid fever (enteria)—the first cure ever found for this too common disease."

Captain Reade reported U. S. Army research teams are "working in happy collaboration" with various divisions of the Institute for Medical Research in Kuala Lumpur. The institute was founded as a tiny one-man laboratory 52 years ago and now includes three large buildings which house divisions dealing with bacteria, viruses, malaria, disease-carrying insects, pathology, biochemistry, nutrition and other research subjects.